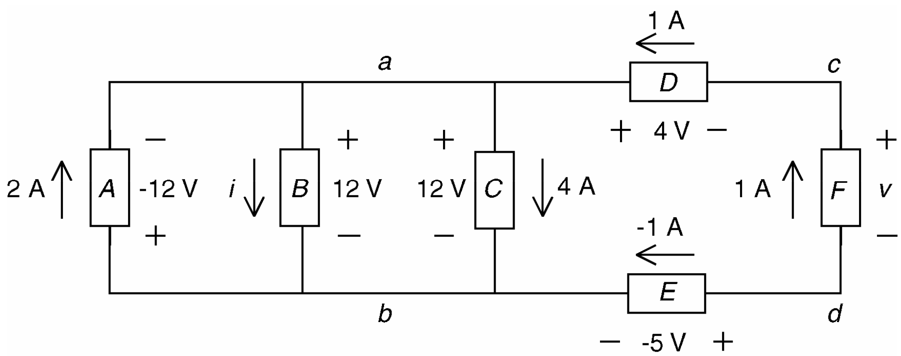
Problems

Section 3-2 Kirchhoff's Laws

P3.2-1



Apply KCL at node *a* to get $2 + 1 = i + 4 \Rightarrow \; i = -1$ A

The current and voltage of element *B* adhere to the passive convention so (12)(-1) = -12 W is power received by element *B*. The power supplied by element *B* is 12 W.

Apply KVL to the loop consisting of elements *D*, *F*, *E*, and *C* to get

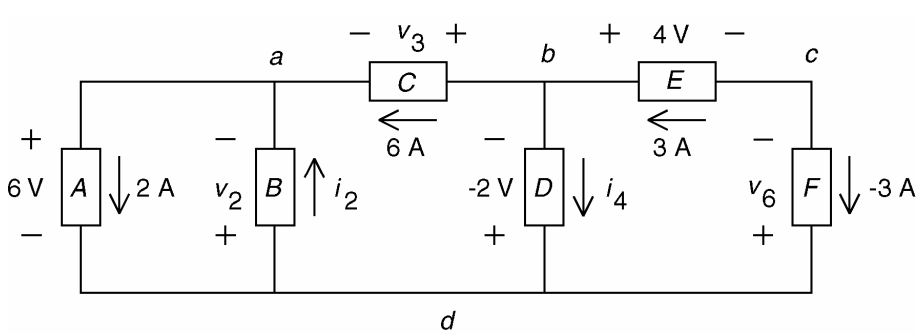
$$4 + v + (-5) - 12 = 0 \Rightarrow \; v = 13 \text{ V}$$

The current and voltage of element *F* do not adhere to the passive convention so (13)(1) = 13 W is the power supplied by element *F*.

Check: The sum of the power supplied by all branches is

$$-(2)(-12) + \underline{12} - (4)(12) + (1)(4) + \underline{13} - (-1)(-5) = 24 + 12 - 48 + 4 + 13 - 5 = 0$$

P3.2-2



Apply KCL at node *a* to get $2 = i_2 + 6 = 0 \Rightarrow \; i_2 = -4$ A

Apply KCL at node b to get $3 = i_4 + 6 \Rightarrow i_4 = -3$ A

Apply KVL to the loop consisting of elements A and B to get

$$-v_2 - 6 = 0 \Rightarrow v_2 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements C, D, and A to get

$$-v_3 - (-2) - 6 = 0 \Rightarrow v_4 = -4 \text{ V}$$

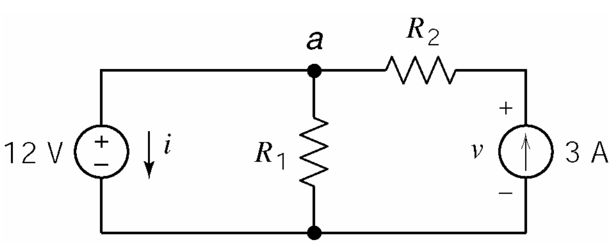
Apply KVL to the loop consisting of elements E, F and D to get

$$4 - v_6 + (-2) = 0 \Rightarrow v_6 = 2 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(6)(2) - (-6)(-4) - (-4)(6) + (-2)(-3) + (4)(3) + (2)(-3) = -12 - 24 + 24 + 6 + 12 - 6 = 0$$

P3.2-3



KVL : $-12 - R_2(3) + v = 0$ (outside loop)

$$v = 12 + 3R_2 \quad \text{or} \quad R_2 = \frac{v-12}{3}$$

KCL $\quad i + \frac{12}{R_1} - 3 = 0$ (top node)

$$i = 3 - \frac{12}{R_1} \quad \text{or} \quad R_1 = \frac{12}{3-i}$$

(a) $v = 12 + 3(3) = 21$ V

$i = 3 - \frac{12}{6} = 1$ A

(b) $R_2 = \frac{2-12}{3} = \underline{-\frac{10}{3}\ \Omega}$; $R_1 = \frac{12}{3-1.5} = \underline{8\ \Omega}$

(checked using LNAP 8/16/02)

(c) $24 = -12\, i$, because 12 and i adhere to the passive convention.

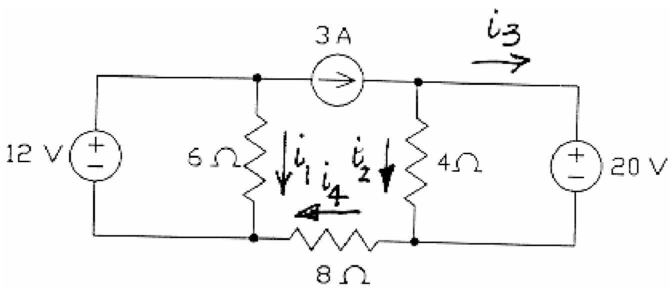
$\therefore \underline{i = -2 \text{ A}}$ and $R_1 = \frac{12}{3+2} = \underline{2.4\ \Omega}$

$9 = 3v$, because 3 and v do not adhere to the passive convention

$\therefore \underline{v = 3 \text{ V}}$ and $R_2 = \frac{3-12}{3} = \underline{-3\ \Omega}$

The situations described in (b) and (c) cannot occur if R_1 and R_2 are required to be nonnegative.

P3.2-4



$$i_1 = \frac{12}{6} = 2\text{ A}$$

$$i_2 = \frac{20}{4} = 5\text{ A}$$

$$i_3 = 3 - i_2 = -2\text{ A}$$

$$i_4 = i_2 + i_3 = 3\text{ A}$$

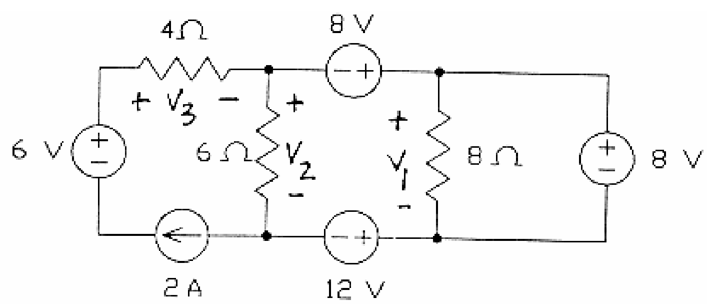
Power absorbed by the 4 Ω resistor = $4 \cdot i_2^2$ = <u>100 W</u>

Power absorbed by the 6 Ω resistor = $6 \cdot i_1^2$ = <u>24 W</u>

Power absorbed by the 8 Ω resistor = $8 \cdot i_4^2$ = <u>72 W</u>

(checked using LNAP 8/16/02)

P3.2-5



$$v_1 = 8\text{ V}$$

$$v_2 = -8 + 8 + 12 = 12\text{ V}$$

$$v_3 = 2 \cdot 4 = 8\text{ V}$$

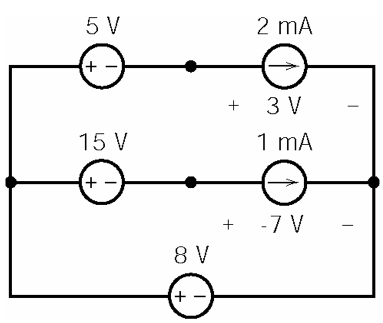
$$4\Omega:\quad P = \frac{v_3^2}{4} = \underline{16\text{ W}}$$

$$6\Omega:\quad P = \frac{v_2^2}{6} = \underline{24\text{ W}}$$

$$8\Omega:\quad P = \frac{v_1^2}{8} = \underline{8\text{ W}}$$

(checked using LNAP 8/16/02)

P3.2-6

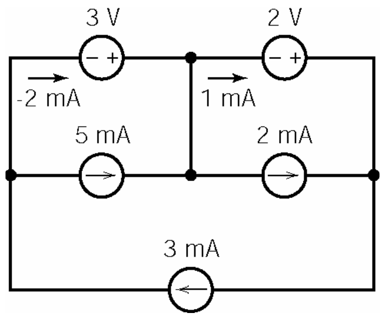


$$P_{2\text{ mA}} = -\left[3 \times \left(2 \times 10^{-3}\right)\right] = -6 \times 10^{-3} = -6\text{ mW}$$

$$P_{1\text{ mA}} = -\left[-7 \times \left(1 \times 10^{-3}\right)\right] = 7 \times 10^{-3} = 7\text{ mW}$$

(checked using LNAP 8/16/02)

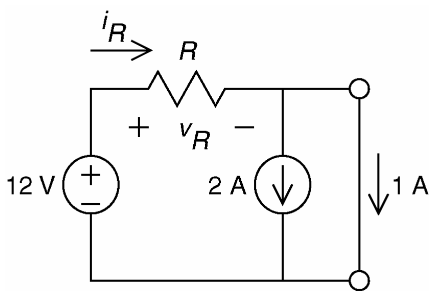
P3.2-7



$$P_{2\text{ V}} = +\left[2\times\left(1\times10^{-3}\right)\right] = 2\times10^{-3} = 2 \text{ mW}$$
$$P_{3\text{ V}} = +\left[3\times\left(-2\times10^{-3}\right)\right] = -6\times10^{-3} = -6 \text{ mW}$$

(checked using LNAP 8/16/02)

P3.2-8



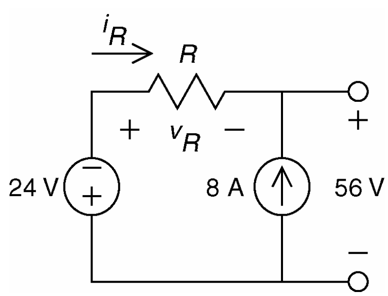
KCL: $i_R = 2+1 \quad\Rightarrow\quad i_R = 3 \text{ A}$

KVL: $v_R + 0 - 12 = 0 \quad\Rightarrow\quad v_R = 12 \text{ V}$

$$\therefore \quad R = \frac{v_R}{i_R} = \frac{12}{3} = 4\,\Omega$$

(checked using LNAP 8/16/02)

P3.2-9



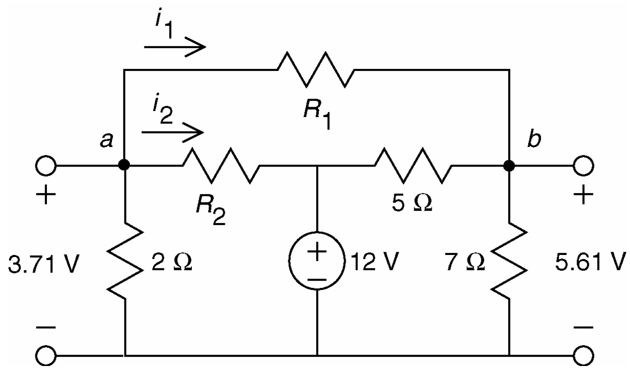
KVL: $v_R + 56 + 24 = 0 \quad\Rightarrow\quad v_R = -80 \text{ V}$

KCL: $i_R + 8 = 0 \quad\Rightarrow\quad i_R = -8 \text{ A}$

$$\therefore \quad R = \frac{v_R}{i_R} = \frac{-80}{-8} = 10\,\Omega$$

(checked using LNAP 8/16/02)

P3.2-10



KCL at node *b*: $$\frac{5.61}{7}=\frac{3.71-5.61}{R_1}+\frac{12-5.61}{5} \Rightarrow 0.801=\frac{-1.9}{R_1}+1.278$$

$$\Rightarrow R_1=\frac{1.9}{1.278-0.801}=3.983\approx 4\,\Omega$$

KCL at node *a*: $$\frac{3.71}{2}+\frac{3.71-5.61}{4}+\frac{3.71-12}{R_2}=0 \Rightarrow 1.855+(-0.475)+\frac{-8.29}{R_2}=0$$

$$\Rightarrow R_2=\frac{8.29}{1.855-0.475}=6.007\approx 6\,\Omega$$

(checked using LNAP 8/16/02)

P3.2-11

The subscripts suggest a numbering of the sources. Apply KVL to get

$$v_1=v_2+v_5+v_9-v_6$$

i_1 and v_1 do not adhere to the passive convention, so

$$p_1=i_1 v_1=i_1\left(v_2+v_5+v_9-v_6\right)$$

is the power supplied by source 1. Next, apply KCL to get

$$i_2=-\left(i_1+i_4\right)$$

i_2 and v_2 do not adhere to the passive convention, so

$$p_2 = i_2 v_2 = -(i_1 + i_4) v_2$$

is the power supplied by source 2. Next, apply KVL to get

$$v_3 = v_6 - (v_5 + v_9)$$

i_3 and v_3 adhere to the passive convention, so

$$p_3 = -i_3 v_3 = -i_3 (v_6 - (v_5 + v_9))$$

is the power supplied by source 3. Next, apply KVL to get

$$v_4 = v_2 + v_5 + v_8$$

i_4 and v_4 do not adhere to the passive convention, so

$$p_4 = i_4 v_4 = i_4 (v_2 + v_5 + v_8)$$

is the power supplied by source 4. Next, apply KCL to get

$$i_5 = i_3 - i_2 = i_3 - (-(i_1 + i_4)) = i_1 + i_3 + i_4$$

i_5 and v_5 adhere to the passive convention, so

$$p_5 = -i_5 v_5 = -(i_1 + i_3 + i_4) v_5$$

is the power supplied by source 5. Next, apply KCL to get

$$i_6 = i_7 - (i_1 + i_3)$$

i_6 and v_6 adhere to the passive convention, so

$$p_6 = -i_6 v_6 = -(i_7 - (i_1 + i_3)) v_6$$

is the power supplied by source 6. Next, apply KVL to get

$$v_7 = -v_6$$

i_7 and v_7 adhere to the passive convention, so

$$p_7 = -i_7\, v_7 = -i_7\left(-v_6\right) = i_7\, v_6$$

is the power supplied by source 7. Next, apply KCL to get

$$i_8 = -i_4$$

i_8 and v_8 do not adhere to the passive convention, so

$$p_8 = i_8\, v_8 = \left(-i_4\right) v_8 = -i_4\, v_8$$

is the power supplied by source 8. Finally, apply KCL to get

$$i_9 = i_1 + i_3$$

i_9 and v_9 adhere to the passive convention, so

$$p_9 = -i_9\, v_9 = -\left(i_1 + i_3\right) v_9$$

is the power supplied by source 9.

(Check: $\sum_{n=1}^{9} p_n = 0$.)

P3.2-12
The subscripts suggest a numbering of the circuit elements. Apply KCL to get

$$i_2 + 0.2 + 0.3 = 0 \quad \Rightarrow \quad i_2 = -0.5 \text{ A}$$

The power received by the 6 Ω resistor is

$$p_2 = 6\, i_2^{\,2} = 6\left(-0.5\right)^2 = 1.5 \text{ W}$$

Next, apply KCL to get

$$i_5 = 0.2 + 0.3 + 0.5 = 1.0 \text{ A}$$

The power received by the 8 Ω resistor is

$$p_5 = 8\, i_5^{\,2} = 8\left(1\right)^2 = 8 \text{ W}$$

Next, apply KVL to get

$$v_7 = 15 \text{ V}$$

The power received by the 20 Ω resistor is

$$p_7 = \frac{v_7^{\,2}}{20} = \frac{15^2}{20} = 11.25 \text{ W}$$

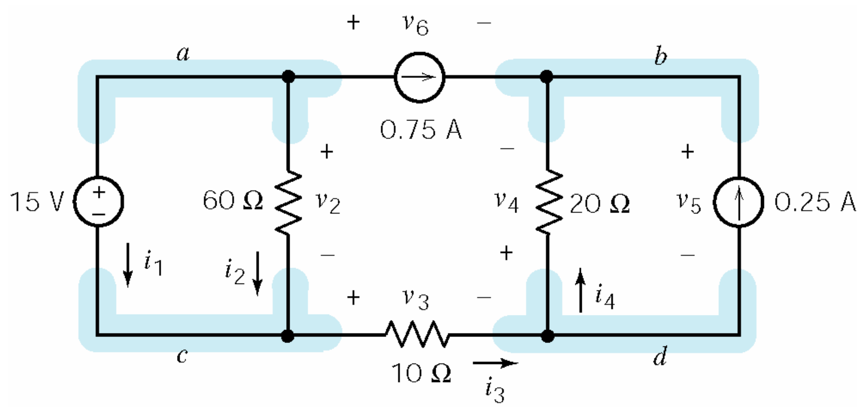
is the power supplied by source 7. Finally, apply KCL to get

$$i_9 = 0.2 + 0.5 = 0.7 \text{ A}$$

The power received by the 5 Ω resistor is

$$p_9 = 5 i_9^{\,2} = 5(0.7)^2 = 2.45 \text{ W}$$

P3.2-13
We can label the circuit as follows:



The subscripts suggest a numbering of the circuit elements. Apply KCL at node *b* to get

$$i_4 + 0.25 + 0.75 = 0 \quad \Rightarrow \quad i_4 = -1.0 \text{ A}$$

Next, apply KCL at node *d* to get

$$i_3 = i_4 + 0.25 = -1.0 + 0.25 = -0.75 \text{ A}$$

Next, apply KVL to the loop consisting of the voltage source and the 60 Ω resistor to get

$$v_2 - 15 = 0 \quad \Rightarrow \quad v_2 = 15 \text{ V}$$

Apply Ohm's law to each of the resistors to get

$$i_2 = \frac{v_2}{60} = \frac{15}{60} = 0.25\ \text{A},$$

$$v_3 = 10\, i_3 = 10(-0.75) = -7.5\ \text{V}$$

and

$$v_4 = 20\, i_4 = 20(-1) = -20\ \text{V}$$

Next, apply KCL at node *c* to get

$$i_1 + i_2 = i_3 \quad \Rightarrow \quad i_1 = i_3 - i_2 = -0.75 - 0.25 = -1.0\ \text{A}$$

Next, apply KVL to the loop consisting of the 0.75 A current source and three resistors to get

$$v_6 - v_4 - v_3 - v_2 = 0 \quad \Rightarrow \quad v_6 = v_4 + v_3 + v_2 = -20 + (-7.5) + 15 = -12.5\ \text{V}$$

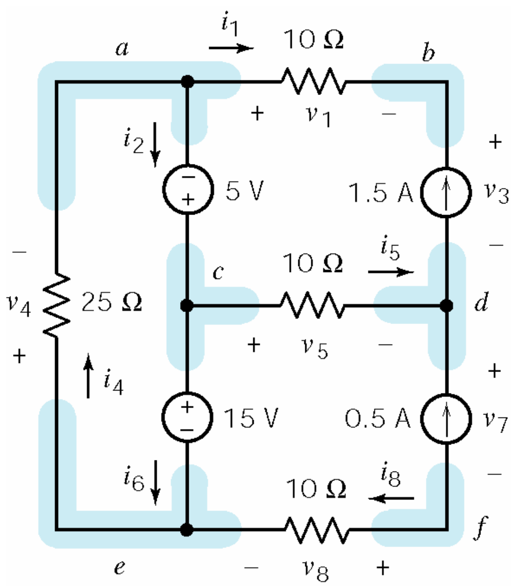
Finally, apply KVL to the loop consisting of the 0.25 A current source and the 20 Ω resistor to get

$$v_5 + v_4 = 0 \quad \Rightarrow \quad v_5 = -v_4 = -(-20) = 20\ \text{V}$$

(Checked: LNAPDC 8/28/04)

P3.2-14

We can label the circuit as follows:



The subscripts suggest a numbering of the circuit elements. Apply KCL at node *b* to get

$$i_1 + 1.5 = 0 \quad \Rightarrow \quad i_1 = -1.5\ \text{A}$$

Apply KCL at node *d* to get

$$i_5 + 0.5 = 1.5 \quad \Rightarrow \quad i_5 = 1.0\ \text{A}$$

Apply KCL at node *f* to get

$$i_8 + 0.5 = 0 \quad \Rightarrow \quad i_8 = -0.5\ \text{A}$$

Apply Ohm's law to each of the 10 Ω resistors to get

$$v_1 = 10\,i_1 = 10(-1.5) = -15 \text{ V}, \quad v_5 = 10\,i_5 = 10(1) = 10 \text{ V} \quad \text{and} \quad v_8 = 10\,i_8 = 10(-0.5) = -5 \text{ V}$$

Apply KVL to the loop consisting of the voltage sources and the 25 Ω resistor to get

$$-5 + 15 + v_4 = 0 \quad \Rightarrow \quad v_4 = -10 \text{ V}$$

Apply Ohm's law to the 25 Ω resistor to get

$$i_4 = \frac{v_4}{25} = \frac{-10}{25} = -0.4 \text{ A}$$

Apply KCL at node *a* to get

$$i_1 + i_2 = i_4 \quad \Rightarrow \quad i_2 = i_4 - i_1 = -0.4 - (-1.5) = 1.1 \text{ A}$$

Apply KCL at node *e* to get

$$i_6 + i_8 = i_4 \quad \Rightarrow i_6 = i_4 - i_8 = -0.4 - (-0.5) = 0.1 \text{ A}$$

Apply KVL to the loop consisting of the 1.5 A current source, the 5 V voltage source and two 10 Ω resistors to get

$$v_1 + v_3 - v_5 + 5 = 0 \quad \Rightarrow \quad v_3 = -5 + v_5 - v_1 = -5 + 10 - (-15) = 20 \text{ V}$$

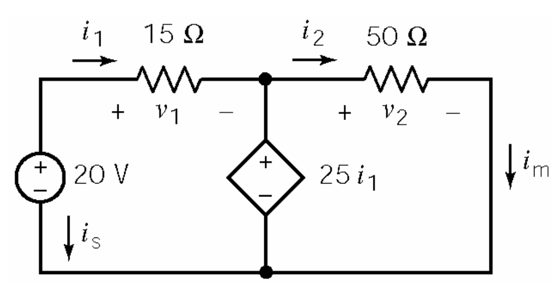
Finally, apply KVL to the loop consisting of the 0.5 A current source, the 15 V voltage source and two 10 Ω resistors to get

$$v_7 + v_8 - 15 + v_5 = 0 \quad \Rightarrow \quad v_7 = 15 - (v_5 + v_8) = 15 - (10 + (-5)) = 10 \text{ V}$$

(Checked: LNAPDC 8/28/04)

P3.2-15

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KVL to node the left mesh to get

$$15 i_1 + 25 i_1 - 20 = 0 \quad \Rightarrow \quad i_1 = \frac{20}{40} = 0.5 \text{ A}$$

Apply KVL to node the left mesh to get

$$v_2 - 25 i_1 = 0 \quad \Rightarrow \quad v_2 = 25 i_1 = 25(0.5) = 12.5 \text{ V}$$

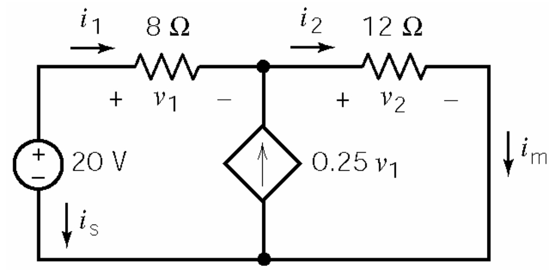
Apply KCL to get $i_m = i_2$. Finally, apply Ohm's law to the 50 Ω resistor to get

$$i_m = i_2 = \frac{v_2}{50} = \frac{12.5}{50} = 0.25 \text{ A}$$

(Checked: LNAPDC 9/1/04)

P3.2-16

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Ohm's law to the 8 Ω resistor to get

$$i_1 = \frac{v_1}{8}$$

Apply KCL at the top node of the CCCS to get

$$i_1 + 0.25 v_1 = i_2 \quad \Rightarrow \quad i_2 = i_1 + 0.25 v_1 = \frac{v_1}{8} + 0.25 v_1 = 0.375 v_1$$

Ohm's law to the 12 Ω resistor to get

$$v_2 = 12 i_2 = 12(0.375 v_1) = 4.5 v_1$$

Apply KVL to the outside loop to get

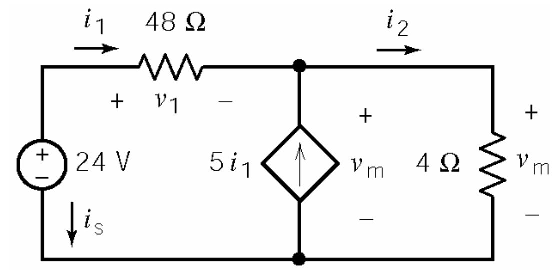
$$v_1 + v_2 - 20 = 0 \quad \Rightarrow \quad v_1 + 4.5 v_1 = 20 \quad \Rightarrow \quad v_1 = \frac{20}{5.5} = 3.636 \text{ V}$$

Apply KCL to get $i_m = i_2$. Finally,

$$i_m = i_2 = 0.375 v_1 = 0.375(3.636) = 1.634 \text{ A}$$

(Checked: LNAPDC 9/1/04)

P3.2-17
We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Ohm's law to the 48 Ω resistor to get

$$v_1 = 48\,i_1$$

Apply KCL at the top node of the CCCS to get

$$i_1 + 5\,i_1 = i_2 \quad \Rightarrow \quad i_2 = 6\,i_1$$

Ohm's law to the 4 Ω resistor to get

$$v_m = 4\,i_2 = 4\left(6\,i_1\right) = 24\,i_1$$

Apply KVL to the outside loop to get

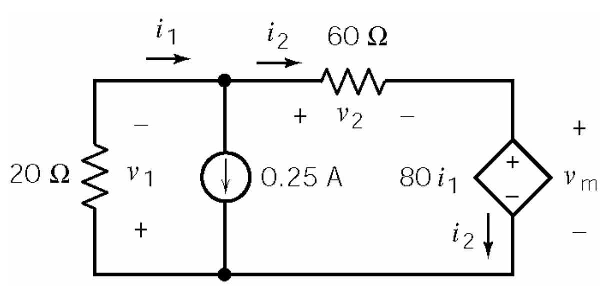
$$v_1 + v_m - 24 = 0 \quad \Rightarrow \quad 48\,i_1 + 24\,i_1 = 24 \quad \Rightarrow \quad i_1 = \frac{24}{72} = \frac{1}{3}\ \text{A}$$

Finally,

$$v_m = 24\,i_1 = 24\left(\frac{1}{3}\right) = 8\ \text{V}$$

(Checked: LNAPDC 9/1/04)

P3.2-18
We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KCL at the top node of the current source to get

$$i_1 = i_2 + 0.25$$

Apply Ohm's law to the resistors to get

$$v_1 = 20\,i_1 \quad \text{and} \quad v_2 = 60\,i_2 = 60\left(i_1 - 0.25\right) = 60\,i_1 - 15$$

Apply KVL to the outside to get

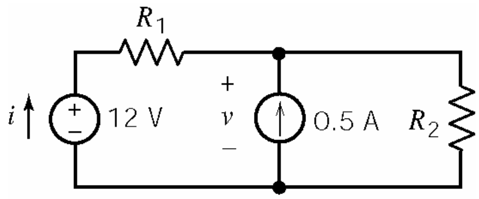
$$v_2 + 80i_1 + v_1 = 0 \quad \Rightarrow \quad (60i_1 - 15) + 80i_1 + 20i_1 = 0 \quad \Rightarrow \quad i_1 = \frac{15}{160} = 0.09375 \text{ A}$$

Finally,

$$v_m = 80i_1 = 80(0.09375) = 7.5 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P3.2-19

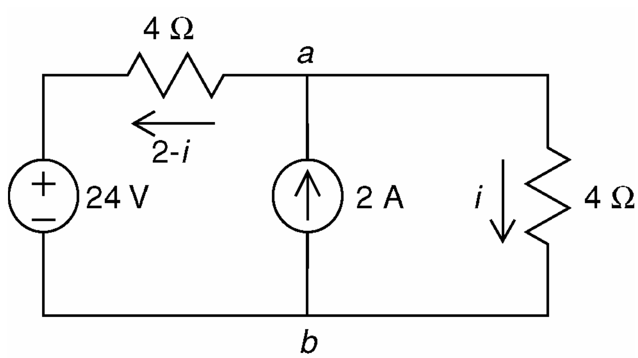


$$i = \frac{4.8}{12} = 0.4 \text{ A} \quad \text{and} \quad v = \frac{3.6}{0.5} = 7.2 \text{ V}$$

$$R_1 = \frac{12 - 7.2}{0.4} = 12\ \Omega \quad \text{and} \quad R_2 = \frac{7.2}{0.4 + 0.5} = 8\ \Omega$$

(Checked: LNAPDC 9/28/04)

P3.2-20



Apply KCL at node a to determine the current in the horizontal resistor as shown.

Apply KVL to the loop consisting of the voltages source and the two resistors to get

-4(2-i) + 4(*i*) - 24 = 0 ⇒ *i* = 4 A

P3.2-21

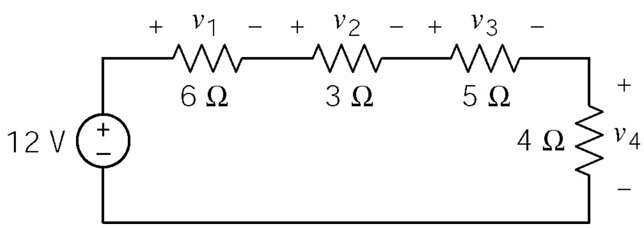
$$-18 + 0 - 12 - v_a = 0 \quad \Rightarrow \quad v_a = -30 \text{ V} \quad \text{and} \quad i_m = \frac{2}{5}v_a + 3 \quad \Rightarrow \quad i_m = 9 \text{ A}$$

P3.2-22

$$-v_a - 10 + 4v_a - 8 = 0 \quad \Rightarrow \quad v_a = \frac{18}{3} = 6 \text{ V} \quad \text{and} \quad v_m = 4v_a = 24 \text{ V}$$

Section 3-3 Series Resistors and Voltage Division

P3.3-1



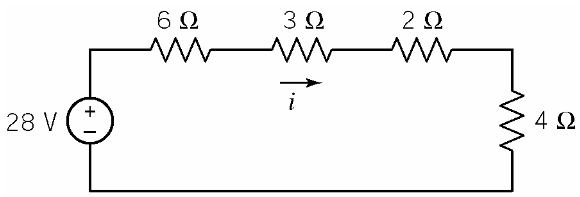
$$v_1 = \frac{6}{6+3+5+4}\,12 = \frac{6}{18}\,12 = \underline{4\text{ V}}$$

$$v_2 = \frac{3}{18}\,12 = \underline{2\text{ V}}\ ;\ v_3 = \frac{5}{18}\,12 = \underline{\frac{10}{3}\text{ V}}$$

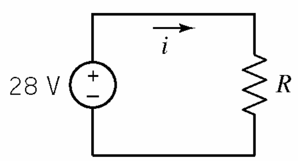
$$v_4 = \frac{4}{18}12 = \underline{\frac{8}{3}\text{ V}}$$

(checked using LNAP 8/16/02)

P3.3-2



(a)



(b)

$(a)\ R = 6+3+2+4 = \underline{15\ \Omega}$

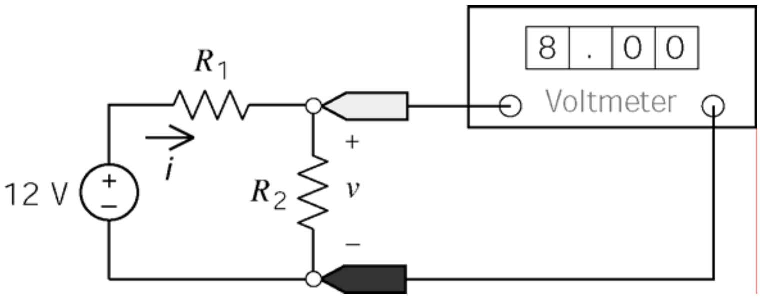
$(b)\ i = \frac{28}{R} = \frac{28}{15} = \underline{1.867\text{ A}}$

$(c)\ p = 28 \cdot i$ =28(1.867)=$\underline{52.27\text{ W}}$

(28 V and i do not adhere to the passive convention.)

(checked using LNAP 8/16/02)

P3.3-3



$$i\,R_2 = v = 8\text{ V}$$
$$12 = i\,R_1 + v = i\,R_1 + 8$$
$$\Rightarrow 4 = i\,R_1$$

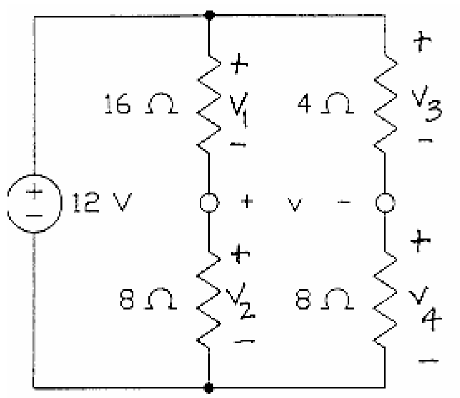
$$(a)\ i = \frac{8}{R_2} = \frac{8}{100}\ ;\ R_1 = \frac{4}{i} = \frac{4\cdot 100}{8} = \underline{50\,\Omega}$$

$$(b)\ i = \frac{4}{R_1} = \frac{4}{100}\ ;\ R_2 = \frac{8}{i} = \frac{8\cdot 100}{4} = \underline{200\,\Omega}$$

$$(c)\ 1.2 = 12\,i \Rightarrow i = 0.1\text{ A}\ ;\ R_1 = \frac{4}{i} = \underline{40\,\Omega};\ R_2 = \frac{8}{i} = \underline{80\,\Omega}$$

(checked using LNAP 8/16/02)

P3.3-4



Voltage division

$$v_1 = \frac{16}{16+8}12 = 8\text{ V}$$

$$v_3 = \frac{4}{4+8}12 = 4\text{ V}$$

KVL: $v_3 - v - v_1 = 0$

$$\underline{v = -4\text{ V}}$$

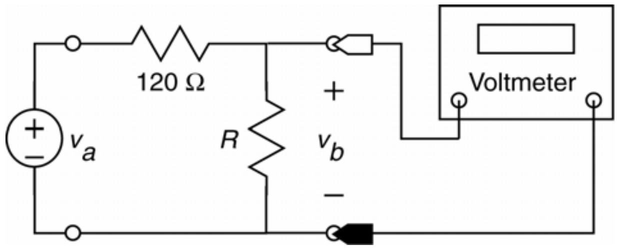
(checked using LNAP 8/16/02)

P3.3-5

using voltage divider: $v_0 = \left(\dfrac{100}{100+2R}\right)v_s \quad \Rightarrow \quad R = 50\left(\dfrac{v_s}{v_o} - 1\right)$

with $v_s = 20$ V and $v_0 > 9$ V, $R < 61.1\,\Omega$
with $v_s = 28$ V and $v_0 < 13$ V, $R > 57.7\,\Omega$
$\Big\}$ $\underline{R = 60\,\Omega}$

P3.3-6



a.) $\left(\frac{240}{120+240}\right)18=12\text{ V}$

b.) $18\left(\frac{18}{120+240}\right)=0.9\text{ W}$

c.) $\left(\frac{R}{R+120}\right)18=2 \quad\Rightarrow\quad 18\,R=2\,R+2\,(120) \quad\Rightarrow\quad R=15\ \Omega$

d.) $0.2=\frac{R}{R+120} \quad\Rightarrow\quad (0.2)(120)=0.8\,R \quad\Rightarrow\quad R=30\ \Omega$

(checked using LNAP 8/16/02)

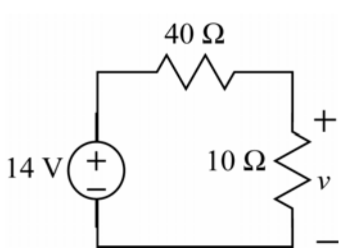
P3.3-7
All of the elements are connected in series.

Replace the series voltage sources with a single equivalent voltage having voltage

$$12 + 20 - 18 = 14\text{ V}.$$

Replace the series 15 Ω, 5 Ω and 20 Ω resistors by a single equivalent resistance of

$$15 + 5 + 20 = 40\ \Omega.$$

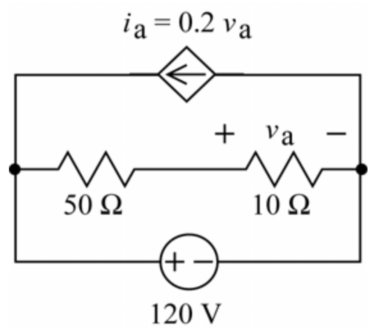


By voltage division

$$v=\left(\frac{10}{10+40}\right)14=\frac{14}{5}=2.8\text{ V}$$

(checked: LNAP 6/9/04)

P3.3-8



Use voltage division to get

$$v_a = \left(\frac{10}{10+50}\right)(120) = 20 \text{ V}$$

Then

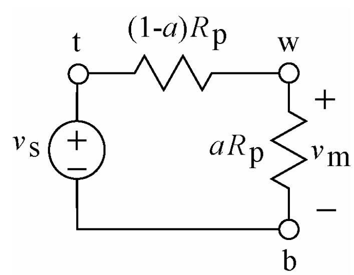
$$i_a = 0.2(20) = 4 \text{ A}$$

The power supplied by the dependent source is given by

$$p = (120)\, i_a = 480 \text{ W}$$

(checked: LNAP 6/21/04)

P3.3-9



(a) Use voltage division to get

$$v_m = \frac{aR_p}{(1-a)R_p + R_p} v_s = a v_s$$

therefore

$$v_m = \left(\frac{v_s}{360}\right)\theta$$

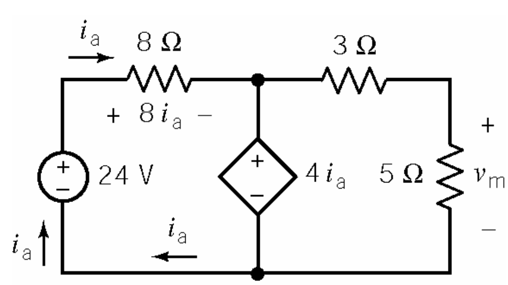
So the input is proportional to the input.

(b) When v_s = 24 V then $v_m = \left(\frac{1}{15}\right)\theta$. When $\theta = 45^\circ$ then v_m = 3 V. When v_m = 10 V then $\theta = 150^\circ$.

(checked: LNAP 6/12/04)

P3.3-10

Replace the (ideal) voltmeter with the equivalent open circuit. Label the voltage measured by the meter. Label some other element voltages and currents.



Apply KVL the left mesh to get

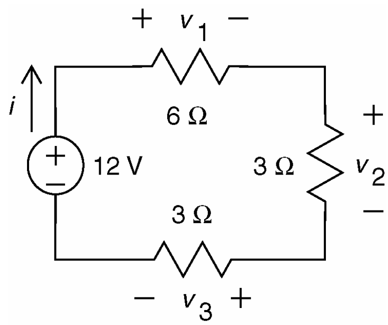
$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

Use voltage division to get

$$v_m = \frac{5}{5+3} 4 i_a = \frac{5}{5+3} 4(2) = 5 \text{ V}$$

(checked using LNAP 9/11/04)

P3.3-11



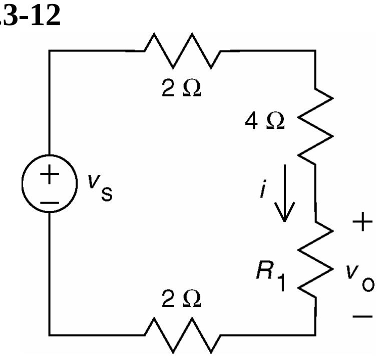
From voltage division $v_3 = 12\left(\frac{3}{3+9}\right) = \underline{3 \text{ V}}$

then $i = \frac{v_3}{3} = \underline{1 \text{ A}}$

The power absorbed by the resistors is: $(1^2)(6)+(1^2)(3)+(1^2)(3)=12$ W

The power supplied by the source is (12)(1) = 12 W.

P3.3-12



$P = 6$ W and $R_1 = 6\ \Omega$

$i^2 = \frac{P}{R_1} = \frac{6}{6} = 1$ or $i = 1$ A

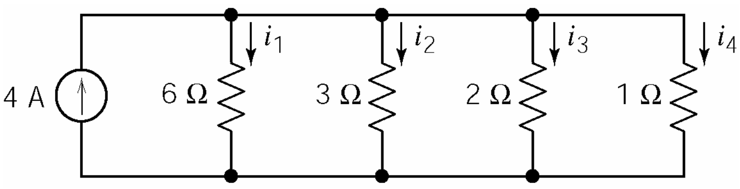
$v_0 = i\, R_1 = (1)\,(6) = \underline{6\text{V}}$

from KVL: $-v_s + i\,(2+4+6+2) = 0$

$\Rightarrow \underline{v_s = 14\, i = 14 \text{ V}}$

Section 3-4 Parallel Resistors and Current Division

P3.4-1



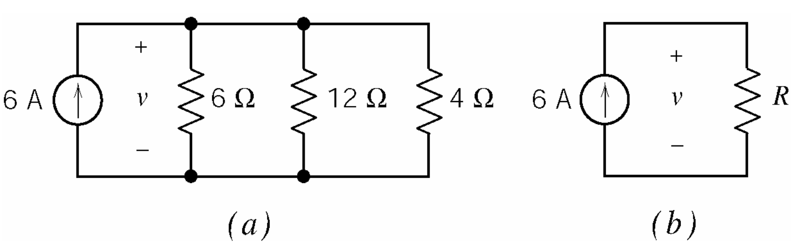
$$i_1 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}4 = \frac{1}{1+2+3+6}\,4 = \underline{\frac{1}{3}\text{ A}}$$

$$i_2 = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}\,4 = \underline{\frac{2}{3}\text{ A}};$$

$$i_3 = \frac{\frac{1}{2}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}\,4 = \underline{1\text{ A}}$$

$$i_4 = \frac{1}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+1}\,4 = \underline{2\text{ A}}$$

P3.4-2

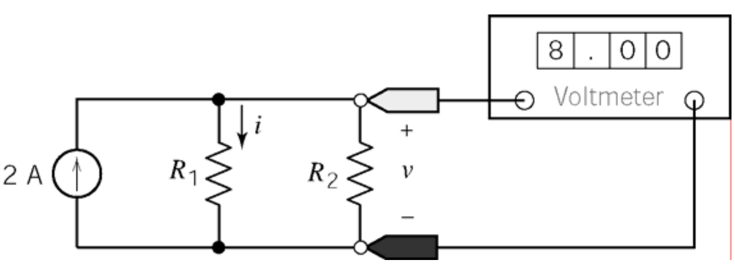


(a) $\frac{1}{R} = \frac{1}{6}+\frac{1}{12}+\frac{1}{4} = \frac{1}{2} \Rightarrow \underline{R\ =\ 2\,\Omega}$

(b) $v = 6\cdot 2 = \underline{12\text{ V}}$

(c) $p = 6\cdot 12 = \underline{72\text{ W}}$

P3.4-3



$$i = \frac{8}{R_1} \text{ or } R_1 = \frac{8}{i}$$

$$8 = R_2(2-i) \Rightarrow i = 2 - \frac{8}{R_2} \text{ or } R_2 = \frac{8}{2-i}$$

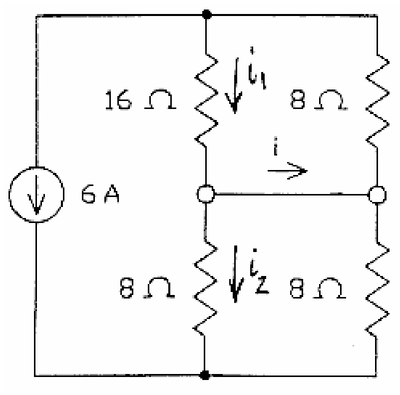
(a) $i = 2 - \frac{8}{12} = \underline{\frac{4}{3}\text{ A}}$; $R_1 = \frac{8}{4/3} = \underline{6\,\Omega}$

(b) $i = \frac{8}{12} = \underline{\frac{2}{3}\text{ A}}$; $R_2 = \frac{8}{2-\frac{2}{3}} = \underline{6\,\Omega}$

$(c)\ R_1 = R_2$ will cause $i = \frac{1}{2}2 = 1$ A. The current in both R_1 and R_2 will be 1 A.

$$2 \cdot \frac{R_1\ R_2}{R_1 + R_2} = 8\ ; \quad R_1 = R_2 \quad \Rightarrow \quad 2 \cdot \frac{1}{2} R_1 = 8 \Rightarrow R_1 = 8 \ \therefore \underline{R_1 = R_2 = 8\,\Omega}$$

P3.4-4



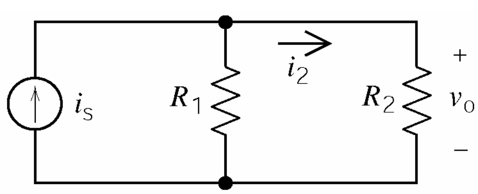
Current division:

$$i_1 = \frac{8}{16+8}(-6) = -2 \text{ A}$$

$$i_2 = \frac{8}{8+8}(-6) = -3 \text{ A}$$

$$i = i_1 - i_2 = \underline{+1 \text{ A}}$$

P3.4-5



current division: $i_2 = \left(\frac{R_1}{R_1 + R_2} \right) i_s$ and

Ohm's Law: $v_o = i_2 R_2$ yields

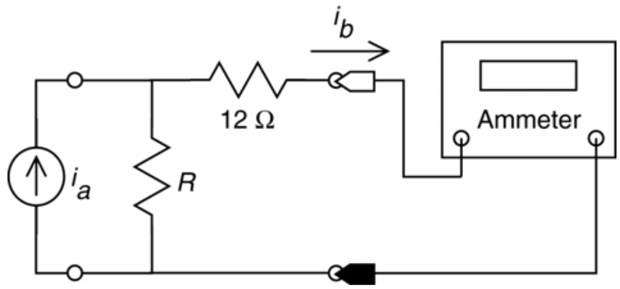
$$i_s = \left(\frac{v_o}{R_2} \right) \left(\frac{R_1 + R_2}{R_1} \right)$$

plugging in $R_1 = 4\Omega$, $v_o > 9$ V gives $i_s > 3.15$ A

and $R_1 = 6\Omega$, $v_o < 13$ V gives $i_s < 3.47$ A

So any $\underline{3.15 \text{ A} < i_s < 3.47 \text{ A}}$ keeps $9 \text{ V} < v_o < 13$ V.

P3.4-6



$$a)\ \left(\frac{24}{12+24}\right)1.8 = 1.2\ \text{A}$$

$$b)\ \left(\frac{R}{R+12}\right)2 = 1.6 \quad\Rightarrow\quad 2\,R = 1.6\,R + 1.6(12) \quad\Rightarrow\quad R = 48\ \Omega$$

$$c)\ 0.4 = \frac{R}{R+12} \quad\Rightarrow\quad (0.4)(12) = 0.6\,R \quad\Rightarrow\quad R = 8\ \Omega$$

P3.4-7

(a) To insure that i_b is negligible we require

$$i_1 = \frac{15}{R_1 + R_2} \geq 10\left(10\times10^{-6}\right) = 10^{-3}$$

So

$$R_1 + R_2 \leq 150\ \text{k}\Omega$$

To insure that the total power absorbed by R_1 and R_2 is no more than 5 mW we require

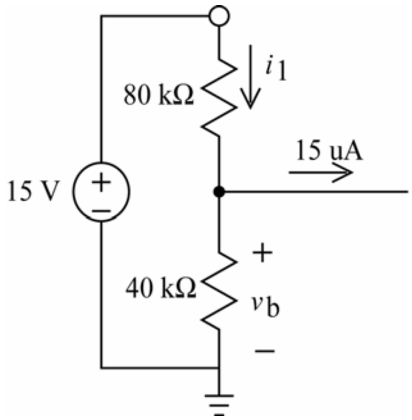
$$\frac{15^2}{R_1 + R_2} \leq 5\times10^{-3} \quad\Rightarrow\quad R_1 + R_2 \geq 45\ \text{k}\Omega$$

Next to cause v_b = 5 V we require

$$5 = v_b = \frac{R_2}{R_1 + R_2}15 \quad\Rightarrow\quad R_1 = 2R_2$$

For example, $R_1 = 40\ \text{k}\Omega,\ R_2 = 80\ \text{k}\Omega,$ satisfy all three requirements.

(b)



KVL gives

$$\left(80\times10^{3}\right)i_{1}+v_{\mathrm{b}}-15=0$$

KCL gives

$$i_{1}=\frac{v_{\mathrm{b}}}{40\times10^{3}}+15\times10^{-6}$$

Therefore

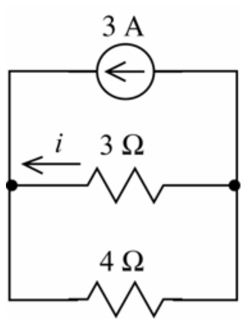
$$\left(80\times10^{3}\right)\left(\frac{v_{\mathrm{b}}}{40\times10^{3}}+15\times10^{-6}\right)+v_{\mathrm{b}}=15$$

Finally

$$3v_{\mathrm{b}}+1.2=15 \quad\Rightarrow\quad v_{\mathrm{b}}=\frac{13.8}{3}=4.6\text{ V}$$

P3.4-8

All of the elements of this circuit are connected in parallel. Replace the parallel current sources by a single equivalent 2 – 0.5 + 1.5 = 3 A current source. Replace the parallel 12 Ω and 6 Ω resistors by a single $\frac{12\times6}{12+6}=4\ \Omega$ resistor.



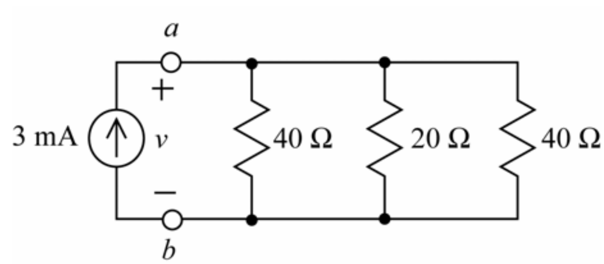
By current division

$$i=\left(\frac{4}{3+4}\right)3=\frac{12}{7}=1.714\text{ A}$$

(checked: LNAP 6/9/04)

P3.4-9

Each of the resistors is connected between nodes *a* and *b*. The resistors are connected in parallel and the circuit can be redrawn like this:



Then

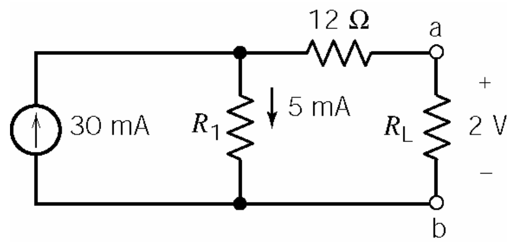
$$40\,||\,20\,||\,40=10\ \Omega$$

So

$$v=10\left(0.003\right)=0.03=30\text{ mV}$$

(checked: LNAP 6/21/04)

P3.4-10



$$R_L = \frac{2}{0.025} = 80\ \Omega$$

$$5\times10^{-3} = \frac{12+R_L}{R_1+(12+R_L)}\left(30\times10^{-3}\right)$$

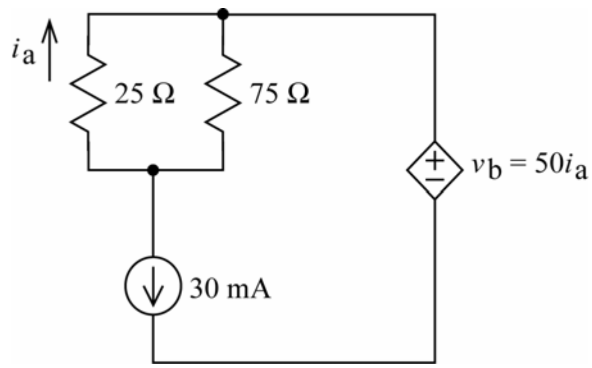
so

$$\frac{1}{6} = \frac{92}{R_1+92} \quad\Rightarrow\quad R_1 = 410\ \Omega$$

(checked: LNAP 6/21/04)

P3.4-11

Use current division to get



$$i_a = -\frac{75}{25+75}\left(30\times10^{-3}\right) = -22.5\text{ mA}$$

So

$$v_b = 50\left(-22.5\times10^{-3}\right) = -1.125\text{ V}$$

The power supplied by the dependent source is given by

$$p = -\left(30\times10^{-3}\right)(-1.125) = 33.75\text{ mW}$$

(checked: LNAP 6/12/04)

P3.4-12

(a) Using current division

$$\frac{20}{R} = \left(\frac{30}{R+30}\right)1 \quad\Rightarrow\quad 20(R+30) = R(30) \quad\Rightarrow\quad R = 60\ \Omega$$

(b) The power supplied by the current source is

$$p = iv = (1)\left[(1)(10)+20\right] = 30\text{ W}$$

P3.4-13

Using voltage division

$$8=\frac{R_1}{R_1+\frac{40R_2}{R_2+40}}\times 24 \quad\Rightarrow\quad \frac{1}{3}=\frac{R_1\left(R_2+40\right)}{R_1R_2+40\left(R_1+R_2\right)}$$

$$\Rightarrow\quad R_1R_2+40\left(R_1+R_2\right)=3R_1R_2+120R_1 \quad\Rightarrow\quad R_1=\frac{40R_2}{2R_2+80}$$

Using KVL

$$24=8+R_2\left(1.6\right) \quad\Rightarrow\quad R_2=10\ \Omega$$

Then

$$R_1=\frac{40\left(10\right)}{2\left(10\right)+80}=4\ \Omega$$

P3.4-14

Using KCL

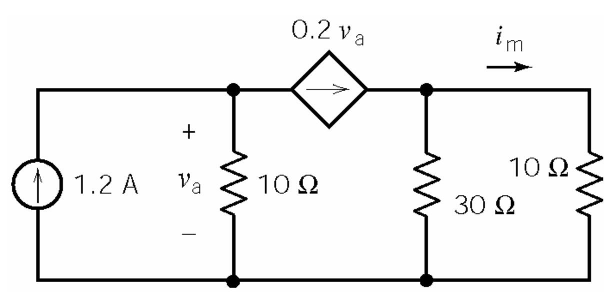
$$.024=0.0192+\frac{0.384}{R_2} \quad\Rightarrow\quad R_2=\frac{0.384}{0.0048}=80\ \Omega$$

Using current division

$$\frac{0.384}{R_2}=\frac{R_1}{R_1+\left(R_2+80\right)}\times 0.024 \quad\Rightarrow\quad 16=\frac{R_1R_2}{R_1+R_2+80}=\frac{80R_1}{R_1+160} \quad\Rightarrow\quad R_1=40\ \Omega$$

P3.4-15

Replace the (ideal) ammeter with the equivalent short circuit. Label the current measured by the meter.



Apply KCL at the left node of the VCCS to get

$$1.2=\frac{v_a}{10}+0.2v_a=0.3v_a \quad\Rightarrow\quad v_a=\frac{1.2}{0.3}=4\ \text{V}$$

Use current division to get

$$i_m=\frac{30}{30+10}\,0.2v_a=\frac{30}{30+10}\,0.2\left(4\right)=0.6\ \text{A}$$

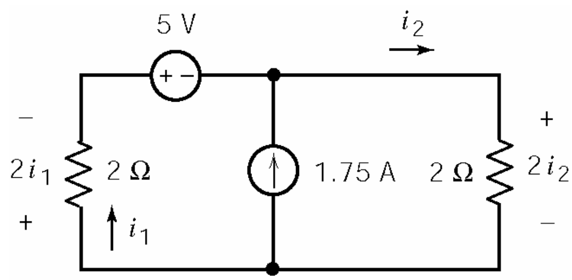
(checked using LNAP 9/11/04)

Section 3-5 Series Voltage Sources and Parallel Current Sources

P3.5-1

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + 2i_1 = 0$$

so

$$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and

$$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

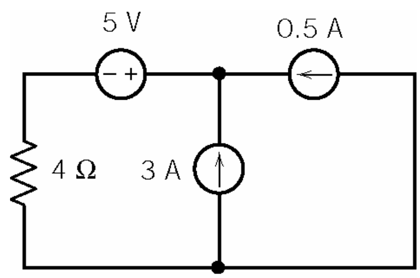
Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

(Checked using LNAP, 9/14/04)

P3.5-2

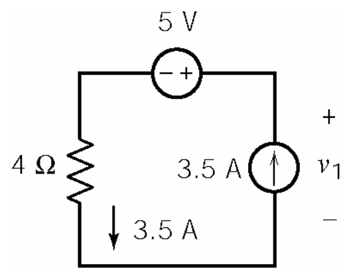
The 20-Ω and 5-Ω resistors are connected in parallel. The equivalent resistance is $\frac{20\times5}{20+5}=4\ \Omega$. The 7-Ω resistor is connected in parallel with a short circuit, a 0-Ω resistor. The equivalent resistance is $\frac{0\times7}{0+7}=0\ \Omega$, a short circuit.

The voltage sources are connected in series and can be replaced by a single equivalent voltage source.



After doing so, and labeling the resistor currents, we have the circuit shown.

The parallel current sources can be replaced by an equivalent current source.



Apply KVL to get

$$-5+v_1-4(3.5)=0 \quad \Rightarrow \quad v_1=19\ \text{V}$$

The power supplied by each sources is:

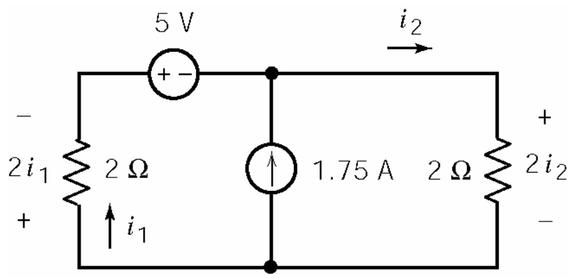
Source	Power delivered
8-V voltage source	$-2(3.5)=-7$ W
3-V voltage source	$-3(3.5)=-10.5$ W
3-A current source	$3\times19=57$ W
0.5-A current source	$0.5\times19=9.5$ W

(Checked using LNAP, 9/15/04)

P3.5-3

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + i_1 = 0$$

so

$$5 + 2\left(i_1 + 1.75\right) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and

$$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

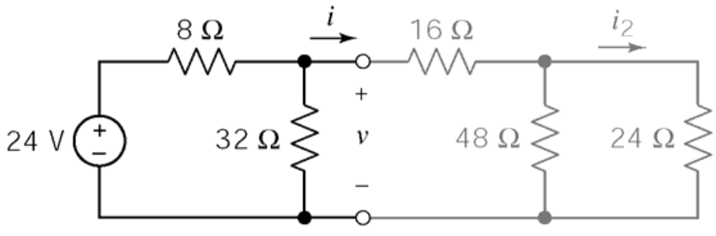
The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

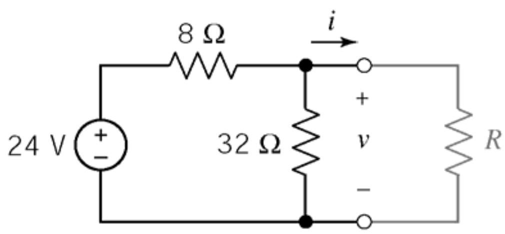
(Checked using LNAP, 9/14/04)

Section 3-6 Circuit Analysis

P3.6-1



(a)



(b)

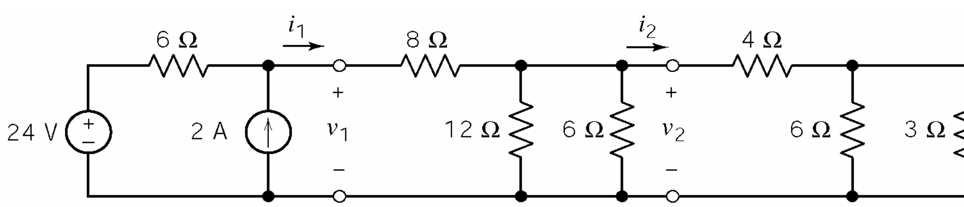
$$(a)\quad R = 16+\frac{48\cdot 24}{48+24} = \underline{32\ \Omega}$$

$$(b)\quad v = \frac{\dfrac{32\cdot 32}{32+32}}{8+\dfrac{32\cdot 32}{32+32}}24 = \underline{16\ \text{V}}\ ;$$

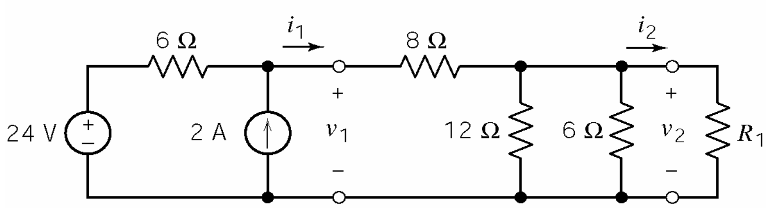
$$i=\frac{16}{32}=\underline{\frac{1}{2}\ \text{A}}$$

$$(c)\quad i_2 = \frac{48}{48+24}\cdot\frac{1}{2} = \underline{\frac{1}{3}\ \text{A}}$$

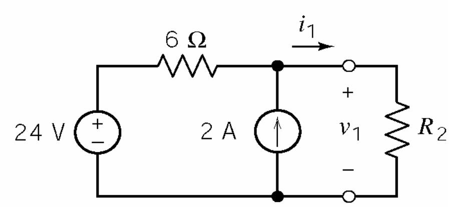
P3.6-2



(a)



(b)



(c)

$$(a)\quad R_1 = 4+\frac{3\cdot 6}{3+6} = \underline{6\,\Omega}$$

$$(b)\quad \frac{1}{R_p} = \frac{1}{12}+\frac{1}{6}+\frac{1}{6} \Rightarrow R_p = 2.4\,\Omega \quad \text{then} \quad R_2 = 8+R_p = \underline{10.4\,\Omega}$$

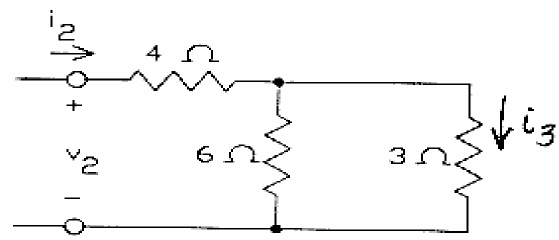
(c) KCL: $i_2 + 2 = i_1$ and $-24 + 6\,i_2 + R_2 i_1 = 0$

$$\Rightarrow \quad -24 + 6\,(i_1 - 2) + 10.4 i_1 = 0$$

$$\Rightarrow \quad i_1 = \frac{36}{16.4} = \underline{2.195 \text{ A}} \quad \Rightarrow \quad v_1 = i_1\, R_2 = 2.2\,(10.4) = \underline{22.83 \text{ V}}$$

$$(d)\ \ i_2 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{12}}\left(2.195\right) = \underline{0.878 \text{ A}},$$

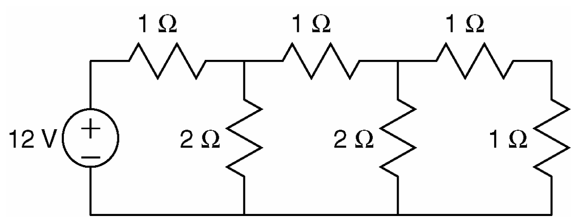
$$v_2 = \left(0.878\right)(6) = \underline{5.3 \text{ V}}$$

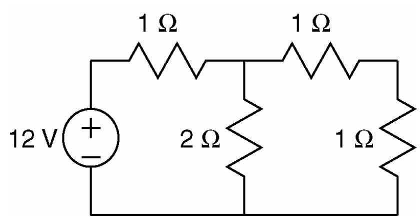


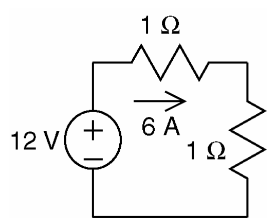
$$(e)\ \ i_3 = \frac{6}{3+6} i_2 = 0.585 \text{ A} \quad \Rightarrow \quad P = 3\, i_3^{\,2} = \underline{1.03 \text{ W}}$$

P3.6-3

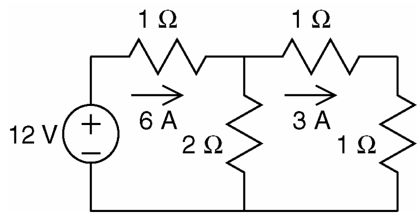
Reduce the circuit from the right side by repeatedly replacing series 1 Ω resistors in parallel with a 2 Ω resistor by the equivalent 1 Ω resistor

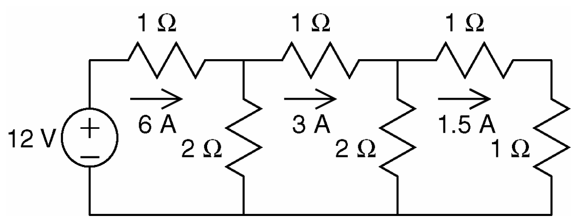


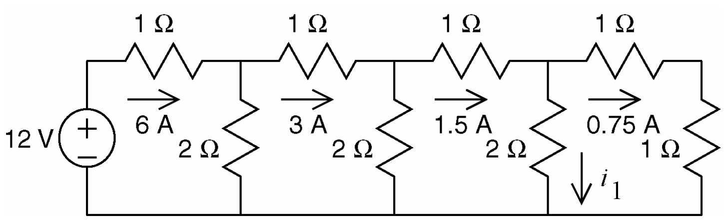




This circuit has become small enough to be easily analyzed. The vertical 1 Ω resistor is equivalent to a 2 Ω resistor connected in parallel with series 1 Ω resistors:







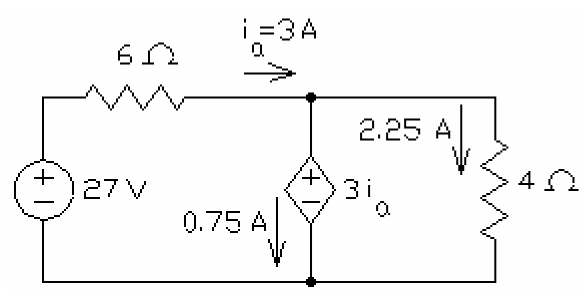
$$i_1 = \frac{1+1}{2+(1+1)}(1.5) = 0.75 \text{ A}$$

P3.6-4

(a)

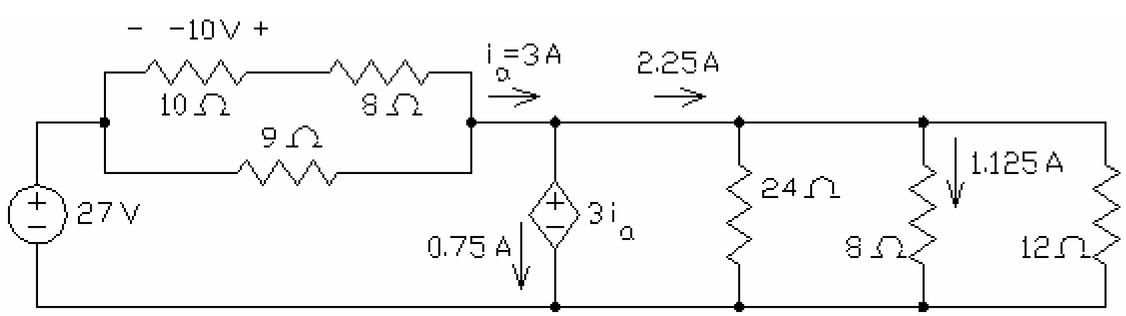
$$\frac{1}{R_2} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8} \Rightarrow R_2 = 4\Omega \quad \text{and} \quad R_1 = \frac{(10+8)\cdot 9}{(10+8)+9} = 6\Omega$$

(b)



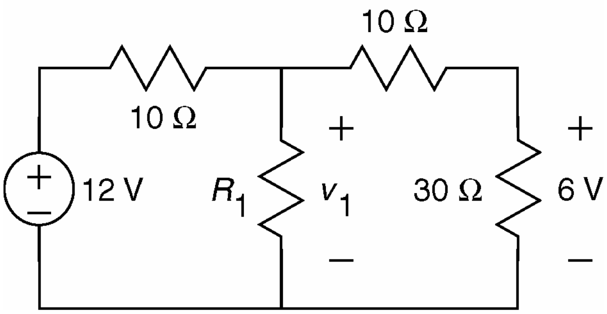
First, apply KVL to the left mesh to get $-27 + 6i_a + 3i_a = 0 \Rightarrow i_a = 3$ A. Next, apply KVL to the left mesh to get $4i_b - 3i_a = 0 \Rightarrow i_b = 2.25$ A.

(c)

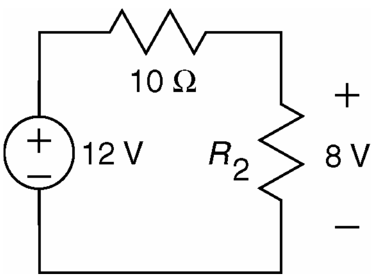


$$i_2 = \frac{\frac{1}{8}}{\frac{1}{24}+\frac{1}{8}+\frac{1}{12}} 2.25 = 1.125 \text{ A} \quad \text{and} \quad v_1 = -(10)\left[\frac{9}{(10+8)+9} 3\right] = -10 \text{ V}$$

P3.6-5



$$\frac{30}{10+30} v_1 = 6 \quad \Rightarrow \quad v_1 = 8 \text{ V}$$



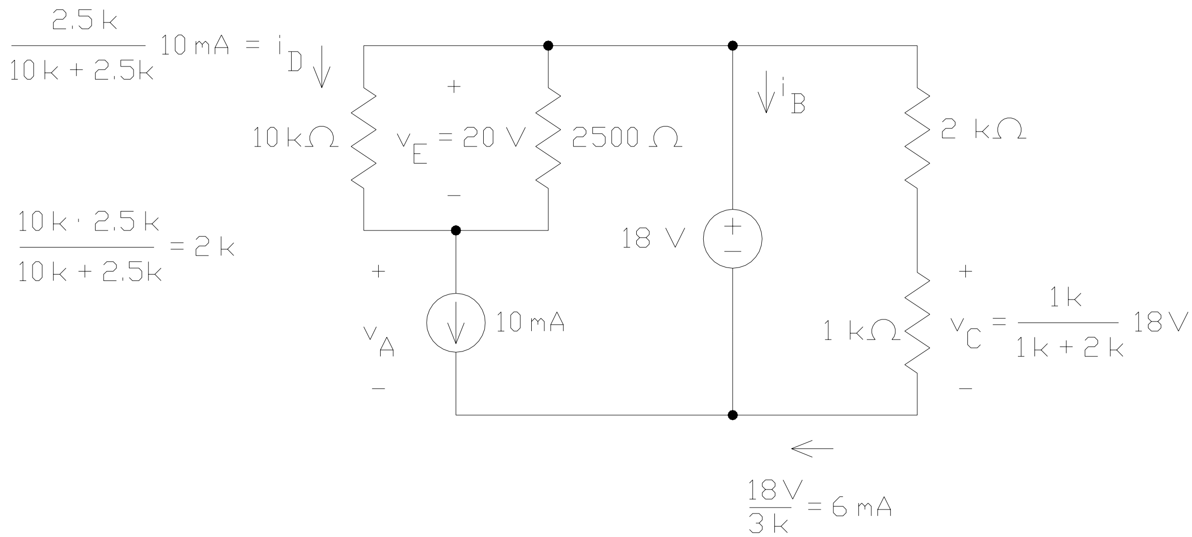
$$\frac{R_2}{R_2+10} 12 = 8 \quad \Rightarrow \quad R_2 = 20\,\Omega$$

$$20 = \frac{R_1(10+30)}{R_1+(10+30)} \quad \Rightarrow \quad R_1 = 40\,\Omega$$

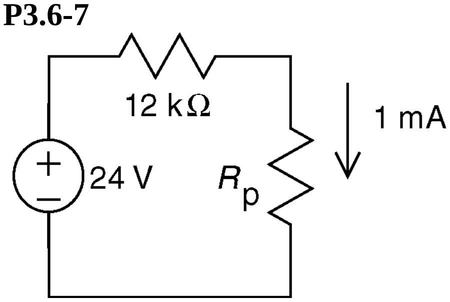
Alternate values that can be used to change the numbers in this problem:

meter reading, V	Right-most resistor, Ω	R_1, Ω
6	30	40
4	30	10
4	20	15
4.8	20	30

P3.6-6



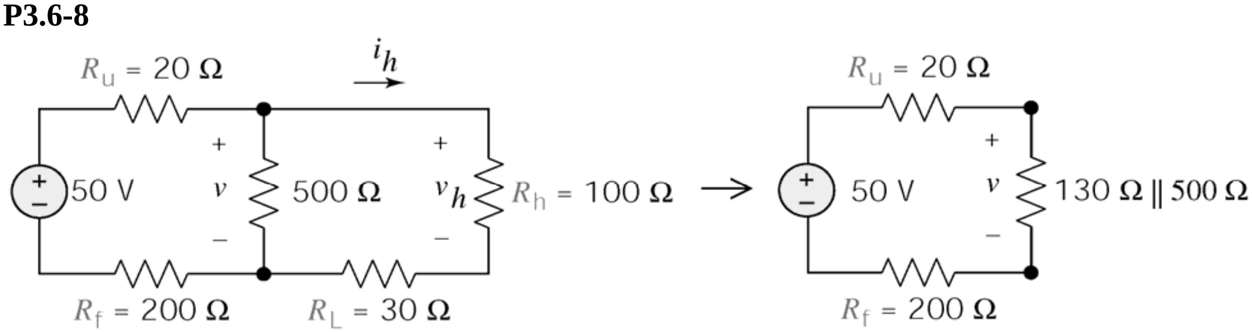
P3.6-7



$$1\times10^{-3} = \frac{24}{12\times10^{3}+R_p} \quad \Rightarrow R_p = 12\times10^{3} = 12\text{ k}\Omega$$

$$12\times10^{3} = R_p = \frac{\left(21\times10^{3}\right)R}{\left(21\times10^{3}\right)+R} \quad \Rightarrow \quad R = 28\text{ k}\Omega$$

P3.6-8

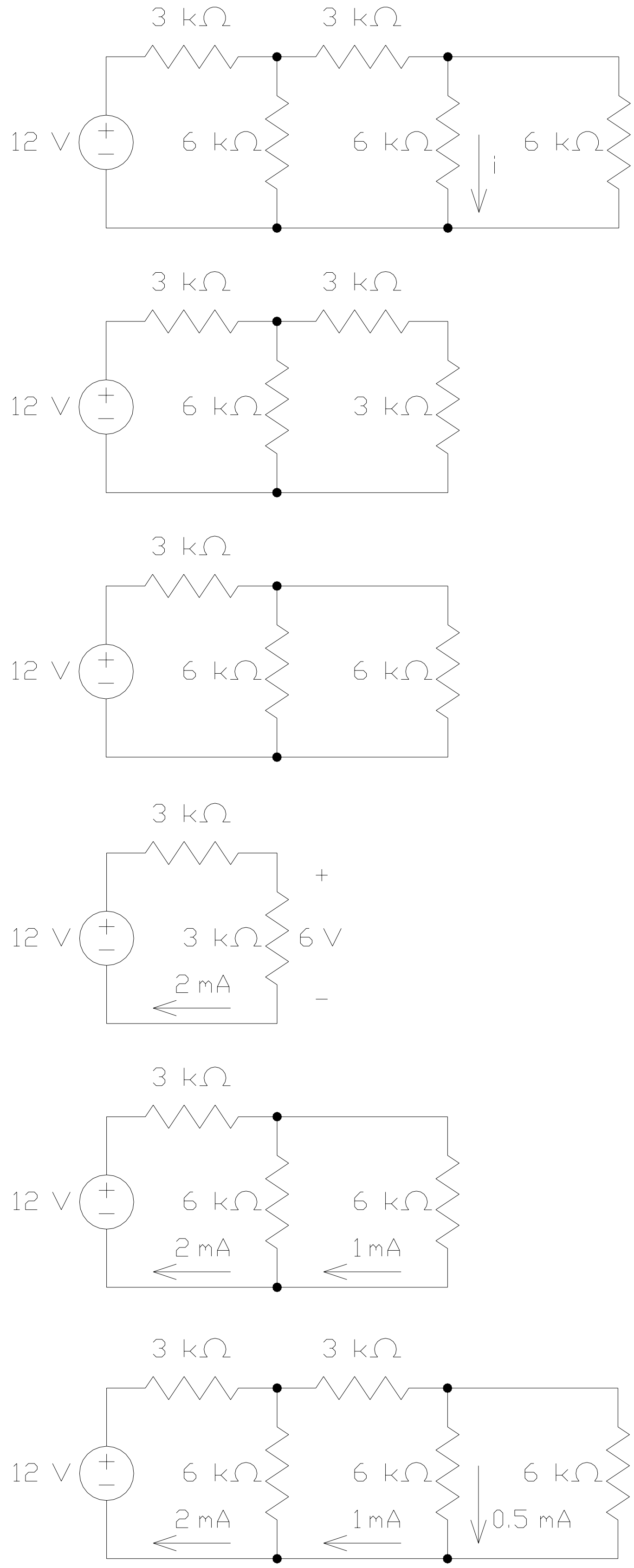


$$\text{Voltage division } \Rightarrow v = 50\left(\frac{130\|500}{130\|500+200+20}\right) = 15.963\ V$$

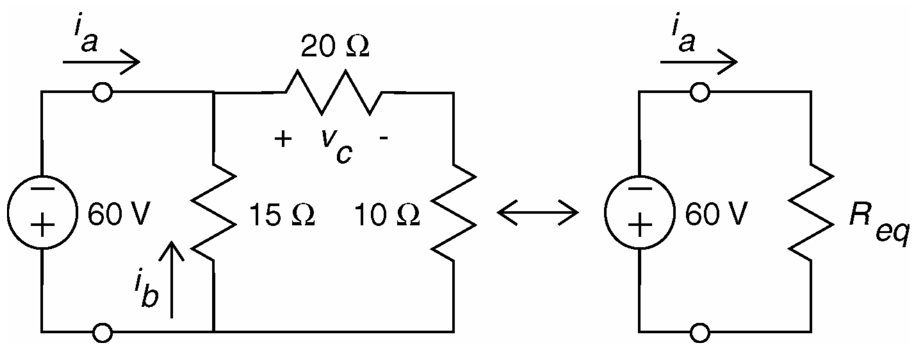
$$\therefore v_h = v\left(\frac{100}{100+30}\right) = (15.963)\left(\frac{10}{13}\right) = \underline{12.279\text{ V}}$$

$$\therefore\ i_h = \frac{v_h}{100} = \underline{.12279\text{ A}}$$

P3.6-9


3 kΩ
3 kΩ
12 V
6 kΩ
6 kΩ
i
6 kΩ
3 kΩ
3 kΩ
12 V
6 kΩ
3 kΩ
3 kΩ
12 V
6 kΩ
6 kΩ
3 kΩ
+
12 V
3 kΩ
6 V
2 mA
−
3 kΩ
12 V
6 kΩ
6 kΩ
2 mA
1 mA
3 kΩ
3 kΩ
12 V
6 kΩ
6 kΩ
6 kΩ
2 mA
1 mA
0.5 mA

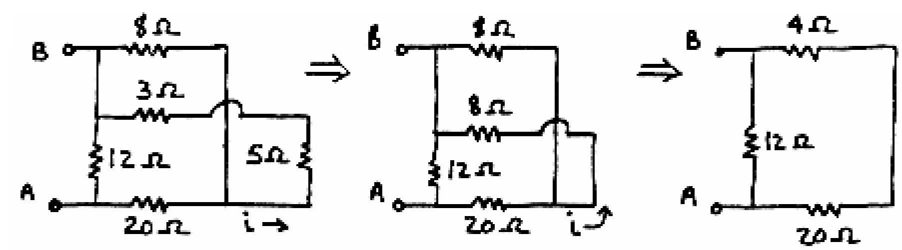
P3.6-10



$$R_{eq} = \frac{15\left(20+10\right)}{15+\left(20+10\right)} = 10\ \Omega$$

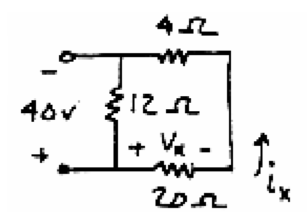
$$i_a = -\frac{60}{R_{eq}} = -6\text{ A},\quad i_b = \left(\frac{30}{30+15}\right)\left(\frac{60}{R_{eq}}\right) = 4\text{ A},\quad v_c = \left(\frac{20}{20+10}\right)\left(-60\right) = -40\text{ V}$$

P3.6-11 **a)**



$$R_{eq} = 24\|12 = \frac{(24)(12)}{24\ +\ 12} = \underline{8\ \Omega}$$

b)

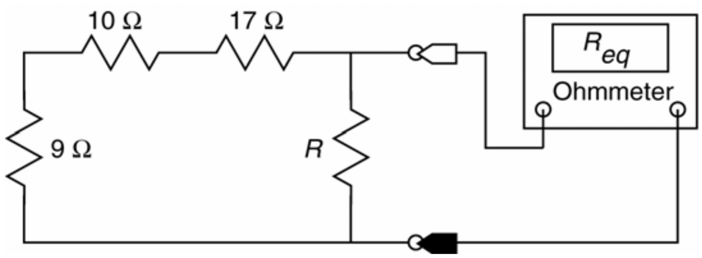


from voltage division:

$$v_x = 40\left(\frac{20}{20+4}\right) = \frac{100}{3}\text{V} \therefore i_x = \frac{\frac{100}{3}}{20} = \frac{5}{3}\text{A}$$

from current division: $i = i_x\left(\frac{8}{8+8}\right) = \underline{\frac{5}{6}\text{ A}}$

P3.6-12

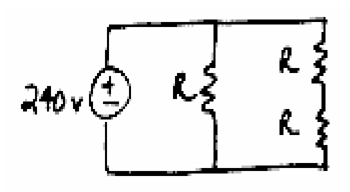


$$9+10+17=36\ \Omega$$

a.) $\dfrac{36(18)}{36+18}=12\ \Omega$

b.) $\dfrac{36\text{ R}}{36+\text{R}}=18 \quad \Rightarrow \quad 18\,R=(18)(36) \quad \Rightarrow \quad R=36\ \Omega$

P3.6-13

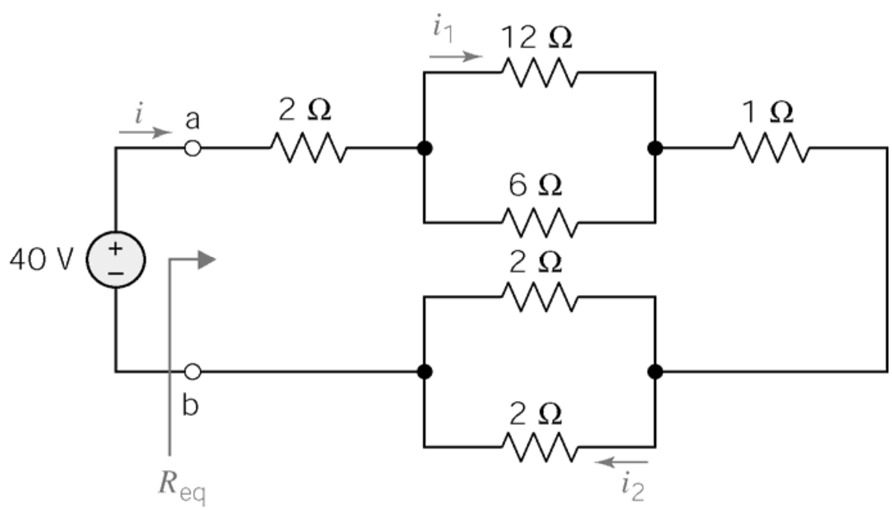


$$R_{eq}=\frac{2R(R)}{2R+R}=\frac{2}{3}R$$

$$P_{\substack{deliv.\\ to\ ckt}}=\frac{v^2}{R_{eq}}=\frac{240}{2/3\,R}=1920\text{ W}$$

Thus $\underline{R=45\ \Omega}$

P3.6-14



$$R_{eq}=2+1+\ (6\|12)+(2\|2)=\ 3+4+1=\underline{8\ \Omega}$$

$$\therefore i=\frac{40}{R_{eq}}=\frac{40}{8}=\underline{5\text{ A}}$$

Using current division

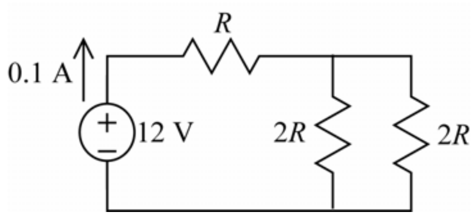
$$i_1=i\left(\frac{6}{6+12}\right)\ =\ (5)\ \left(1/3\right)\ =\underline{5/3\text{ A}} \quad \text{and} \quad i_2=i\left(\frac{2}{2+2}\right)=(5)\left(1/2\right)=\underline{5/2\text{ A}}$$

P3.6-15

$$\left(R \,||\, 4R\right) + \left(2R \,||\, 3R\right) = \frac{4}{5}R + \frac{6}{5}R = 2R$$

$$R + \left(2R \,||\, \left(R + \left(2R \,||\, 2R\right)\right)\right) = R + \left(2R \,||\, 2R\right) = 2R$$

So the circuit is equivalent to



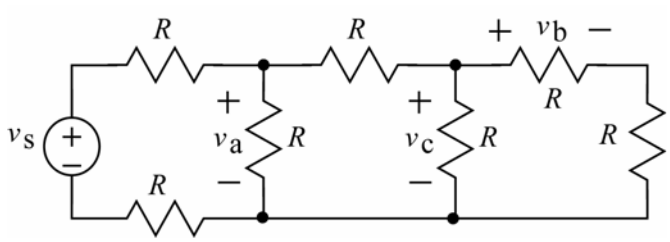
Then

$$12 = 0.1\left(R + \left(2R \,||\, 2R\right)\right) = 0.1\left(2R\right) \quad \Rightarrow \quad R = 60\ \Omega$$

(checked: ELAB 5/31/04)

P3.6-16

The circuit can be redrawn as



$$v_a = \frac{R \,||\, \left(R + \left(R \,||\, 2R\right)\right)}{2R + R \,||\, \left(R + \left(R \,||\, 2R\right)\right)} v_s = \frac{5}{21} v_s$$

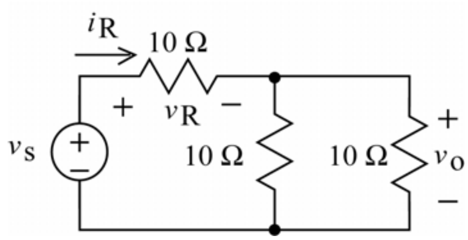
$$v_c = \frac{R \,||\, 2R}{R + \left(R \,||\, 2R\right)} v_s = \frac{2}{5} v_a = \frac{2}{21} v_s$$

$$v_b = \frac{R}{R + R} v_c = \frac{1}{2} v_c = \frac{1}{21} v_s$$

(Checked using LNAP 5/23/04)

P3.6-17

$$v_o = \frac{(10 \| 10)}{10 + (10 \| 10)} v_s = \frac{5}{15} v_s = \frac{v_s}{3}$$



$$v_R + v_o - v_s = 0 \quad \Rightarrow \quad v_R = \frac{2}{3} v_s$$

$$i_R = \frac{v_R}{10} = \frac{2}{30} v_s$$

$$P = \left(\frac{2}{30} v_s\right)^2 (10) = \frac{4}{90} v_s^{\,2} \le \frac{1}{4} \quad \Rightarrow \quad |v_s| \le \sqrt{\frac{90}{16}} = \frac{3\sqrt{10}}{4} = 2.37 \text{ V}$$

(checked: LNAP 5/31/04)

P3.6-18

The voltage across each strain gauge is $\frac{v_s}{2}$ so the current in each strain gauge is $\frac{v_s}{240}$.

$$0.2 \times 10^{-3} \ge \frac{v_s^{\,2}}{480} \quad \Rightarrow \quad |v_s| \le \sqrt{96 \times 10^{-3}} = 0.31 \text{ V}$$

(checked: LNAP 6/9/04)

P3.6-19

(a)

$$R_1 = 10 \| (30 + 10) = 8 \ \Omega$$
$$R_2 = 4 + (18 \| 9) = 10 \ \Omega$$
$$R_3 = 6 \| (6 + 6) = 4 \ \Omega$$

(b)

$$i = 1 \text{ A}$$
$$v_1 = 8 \text{ V}, \ v_2 = 4 \text{ V}$$

(c)

$$v_4 = -\frac{10}{10+30} 8 = -2 \text{ V}$$

$$i_5 = -\frac{9}{9+18} 1 = -\frac{1}{3} \text{ A}$$

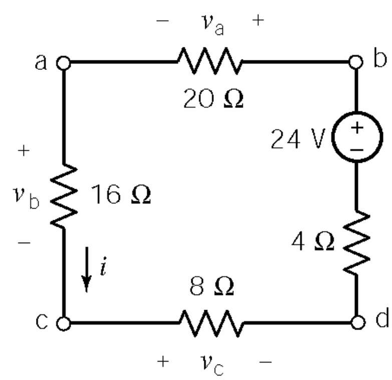
$$v_7 = -18\left(-\frac{1}{3}\right) = +6 \text{ V}$$

$$i_6 = \frac{4}{12} = \frac{1}{3} \text{ A}$$

(checked: LNAP 6/6/04)

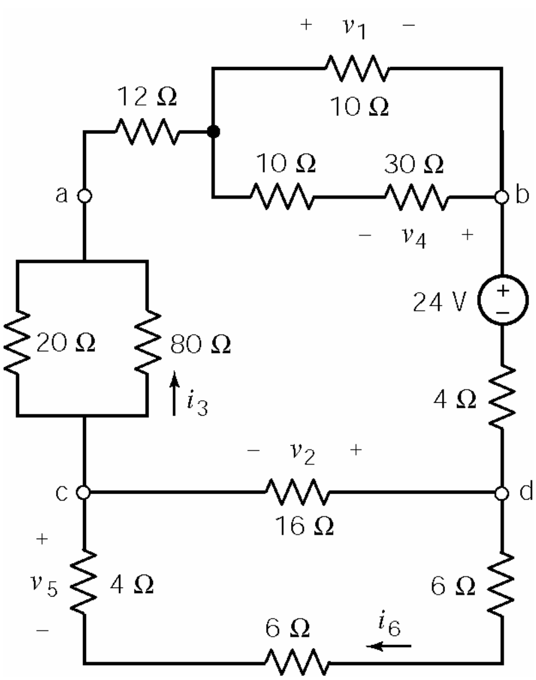
P3.6-20

Replace series and parallel combinations of resistances by equivalent resistances. Then KVL gives



$$\left(20+4+8+16\right) i = 48 \quad \Rightarrow \quad i = 0.5 \text{ A}$$

$$v_a = 20i = 10 \text{ V},\ v_b = 16i = 8 \text{ V and } v_c = 8i = 4 \text{ V}$$



Compare the original circuit to the equivalent circuit to get

$$v_1 = -\left(\frac{10 \,||\, (10+30)}{12+10 \,||\, (10+30)}\right) v_a = -\left(\frac{8}{12+8}\right) 10 = -4 \text{ V}$$

$$v_2 = -v_c = -4 \text{ V}$$

$$i_3 = -\left(\frac{20}{20+80}\right) i = -\left(\frac{1}{5}\right)(0.5) = -0.1 \text{ A}$$

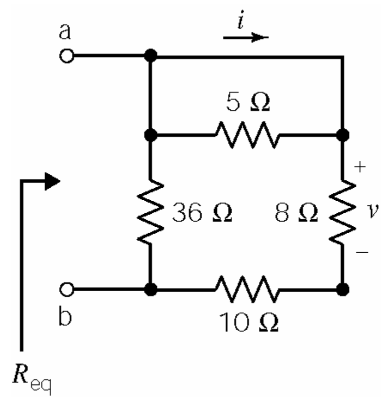
$$v_4 = -\left(\frac{30}{10+30}\right) v_1 = -\left(\frac{1}{4}\right)(-4) = 1 \text{ V}$$

$$v_5 = \left(\frac{4}{5+6+6}\right) v_c = \left(\frac{1}{4}\right)(4) = 1 \text{ V}$$

$$i_6 = -\left(\frac{16}{16+(4+6+6)}\right) i = -\left(\frac{1}{2}\right)(0.5) = -0.25 \text{ A}$$

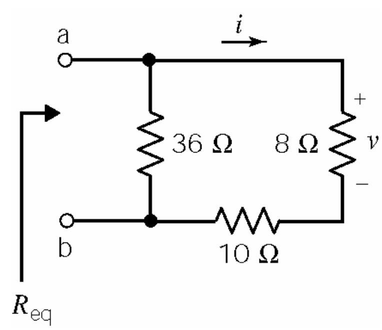
(checked: LNAP 6/10/04)

P3.6-21



Replace parallel resistors by equivalent resistors:

$$6 \parallel 30 = 5\ \Omega \text{ and } 72 \parallel 9 = 8\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.

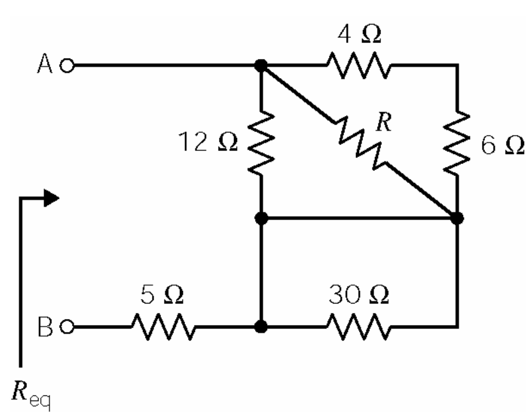
$$R_{eq} = 36 \parallel (8+10) = 12\ \Omega$$

$$v = \frac{8}{8+10} v_{ab} = \frac{4}{9}(18) = 8 \text{ V}$$

$$i = \frac{v}{8} = 1 \text{ A}$$

(checked: LNAP 6/21/04)

P3.6-22



Replace parallel resistors by an equivalent resistor:

$$8 \parallel 24 = 6\ \Omega$$

A short circuit in parallel with a resistor is equivalent to a short circuit.

Replace series resistors by an equivalent resistor:

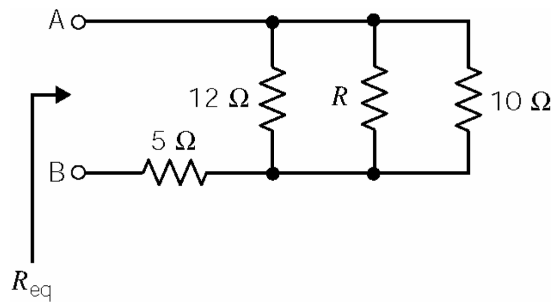
$$4+6 = 10\ \Omega$$

Now

$$9 = R_{\text{eq}} = 5 + (12 \,\|\, R \,\|\, 10)$$

so

$$4 = \frac{R \times \frac{60}{11}}{R + \frac{60}{11}} \quad \Rightarrow \quad R = 15\ \Omega$$



(checked: LNAP 6/21/04)

P3.6-23

$$R_{\text{eq}} = \left(R \,\|\, (R+R) \,\|\, R\right) \,\|\, \left(R \,\|\, (R+R) \,\|\, R\right)$$

$$R \,\|\, (R+R) \,\|\, R = 2R \,\|\, \frac{R}{2} = \frac{2}{5} R$$

$$R_{\text{eq}} = \frac{2}{5} R \,\|\, \frac{2}{5} R = \frac{R}{5} \quad \Rightarrow \quad R = 5\, R_{\text{eq}} = 250\ \Omega$$

(checked: LNAP 6/21/04)

P3.6-24

$$i_{\text{a}} = \frac{9.74}{8} = 1.2175\ \text{A}$$

$$9.74 - 6.09 = r\, i_{\text{a}} = r\left(\frac{9.74}{8}\right) \quad \Rightarrow \quad r = \left(\frac{9.74\text{-}6.09}{9.74}\right) 8 = 3\ \frac{\text{V}}{\text{A}}$$

$$v_{\text{b}} = 12 - 9.74 = 2.26\ \text{V}$$

$$g v_{\text{b}} + \frac{6.09}{8} + \frac{9.74}{8} - \frac{2.26}{8} = 0 \quad \Rightarrow \quad g v_{\text{b}} = -1.696\ \text{A}$$

$$g = \frac{g v_{\text{b}}}{v_{\text{b}}} = \frac{-6.696}{2.26} = -0.75$$

(checked: LNAP 6/21/04)

P3.6-25

$$v_a = \frac{20 \parallel 20}{20+(20 \parallel 20)} v_s = \frac{1}{3} v_s$$

$$v_o = \left(\frac{12}{12+8}\right)(10 v_a) = \frac{3}{5} \times 10 \times \frac{1}{3} v_s = 2 v_s$$

So v_o is proportional to v_s and the constant of proportionality is $2 \ \frac{\text{V}}{\text{V}}$.

P3.6-26

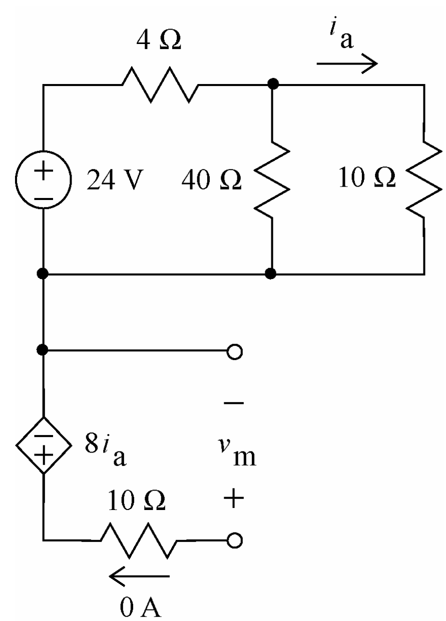
$$i_a = \left(\frac{40}{40+10}\right) \frac{v_s}{2+(40 \parallel 10)} = \left(\frac{4}{5}\right)\left(\frac{v_s}{10}\right) = \frac{4}{50} v_s$$

$$i_o = -\left(\frac{40}{20+40}\right)(50 i_a) = -\frac{100}{3}\left(\frac{4}{50}\right) v_s = -\frac{8}{3} v_s$$

The output is proportional to the input and the constant of proportionality is $-\frac{8}{3} \ \frac{\text{A}}{\text{V}}$.

P3.6-27

Replace the voltmeter by the equivalent open circuit and label the voltage measured by the meter as v_m.



The 10-Ω resistor at the right of the circuit is in series with the open circuit that replaced the voltmeter so it's current is zero as shown. Ohm's law indicates that the voltage across that 10-Ω resistor is also zero. Applying KVL to the mesh consisting of the dependent voltage source, 10-Ω resistor and open circuit shows that

$$v_m = 8\, i_a$$

The 10-Ω resistor and 40-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

$$\frac{40 \times 10}{40+10} = 8\ \Omega$$

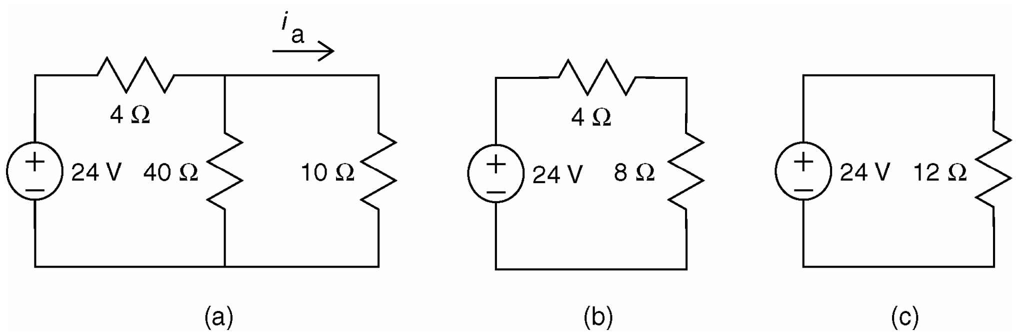
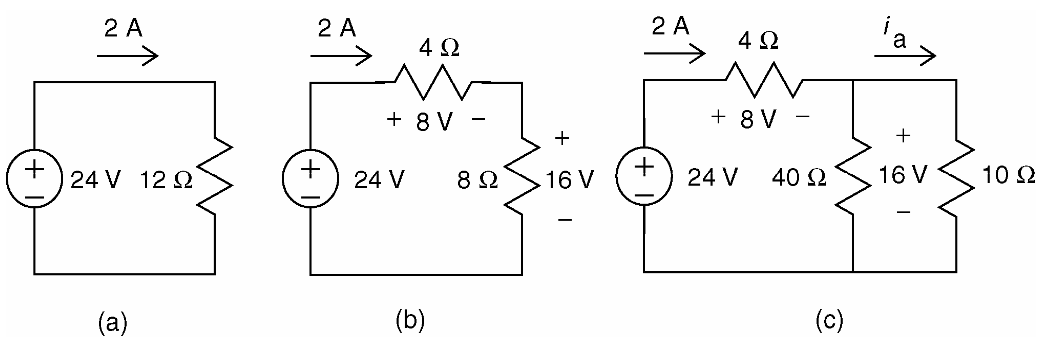


Figure a shows part of the circuit. In Figure b, an equivalent resistor has replaced the parallel resistors. Now the 4-Ω resistor and 8-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single resistor with a resistance equal to $4+8=12\ \Omega$. In Figure c, an equivalent resistor has replaced the series resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



Consider Figure a. Using Ohm's law, we see that the current in the 12-Ω resistor is 2 A. The current in the voltage source is also 2 A. Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the voltage source must also be 2 A in Figure b. The currents in resistors in Figure b are equal to the current in the voltage source. Next, Ohm's law is used to calculate the resistor voltages as shown in Figure b.

Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the 4-Ω resistor in Figure c must be equal to the current in the 4-Ω resistor in Figure b. Using current division in Figure c are yields

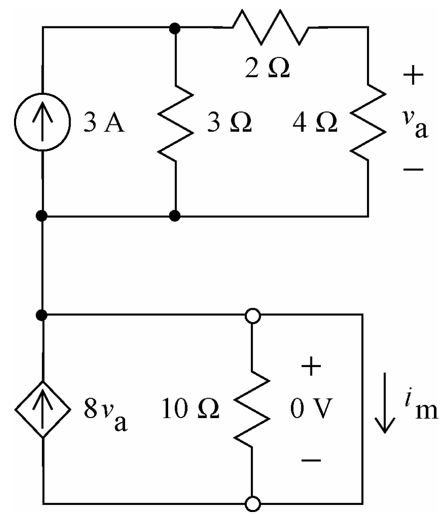
$$i_a = \left(\frac{40}{40+10}\right) 2 = 1.6\ \text{A}$$

Finally,

$$v_m = 8\, i_a = 8 \times 1.6 = 12.8\ \text{V}$$

P3.6-28

Replace the ammeter by the equivalent short circuit and label the current measured by the meter as i_m.

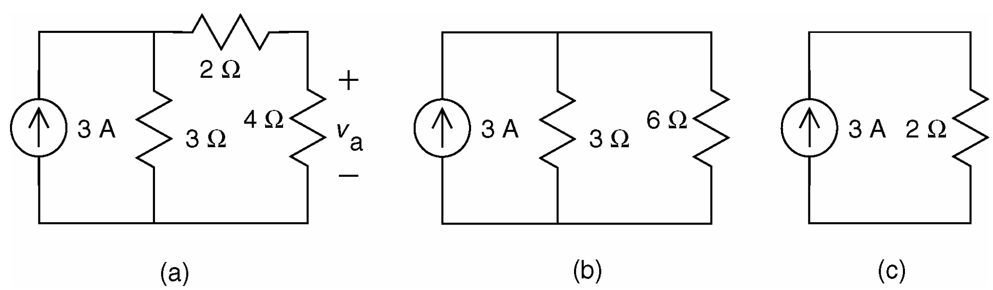


The 10-Ω resistor at the right of the circuit is in parallel with the short circuit that replaced the ammeter so it's voltage is zero as shown. Ohm's law indicates that the current in that 10-Ω resistor is also zero. Applying KCL at the top node of that 10-Ω resistor shows that

$$i_m = 0.8\, v_a$$

Figure a shows part of the circuit. The 2-Ω resistor and 4-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single resistor with a resistance equal to

$$2 + 4 = 6\ \Omega$$



P3.6-29

Use current division in the top part of the circuit to get

$$i_a = \left(\frac{40}{40+10}\right)(-3) = -2.4\ \text{ A}$$

Next, denote the voltage measured by the voltmeter as v_m and use voltage division in the bottom part of the circuit to get

$$v_m = \left(\frac{R}{18+R}\right)(-5\, i_a) = \left(\frac{-5\, R}{18+R}\right) i_a$$

Combining these equations gives:

$$v_m = \left(\frac{-5\, R}{18+R}\right)(-2.4) = \frac{12\, R}{18+R}$$

When $v_m = 4$ V,

$$4 = \frac{12\,R}{18+R} \quad \Rightarrow \quad R = \frac{4\times 18}{12-4} = 9\ \Omega$$

P3.6-30

Use voltage division in the top part of the circuit to get

$$v_a = \left(\frac{12}{12+18}\right)\left(-v_s\right) = -\frac{2}{5}\,v_s$$

Next, use current division in the bottom part of the circuit to get

$$i_m = -\left(\frac{16}{16+R}\right)\left(5\,v_a\right) = \left(-\frac{80}{16+R}\right)v_a$$

Combining these equations gives:

$$i_m = \left(-\frac{80}{16+R}\right)\left(-\frac{2}{5}v_s\right) = \left(\frac{32}{16+R}\right)v_s$$

a. When $v_s = 15$ V and $i_m = 5$ A

$$5 = \left(\frac{32}{16+R}\right)15 \quad \Rightarrow \quad 80 + 5\,R = 480 \quad \Rightarrow \quad R = \frac{400}{5} = 80\ \Omega$$

b. When $v_s = 15$ V and $R = 24\ \Omega$

$$i_m = \left(\frac{32}{16+24}\right)15 = 12\ \text{A}$$

c. When $i_m = 3$ A and $R = 24\ \Omega$

$$3 = \left(\frac{32}{16+24}\right)v_s = \frac{4}{5}v_s \quad \Rightarrow \quad v_s = \frac{15}{4} = 3.75\ \text{V}$$

P3.6-31

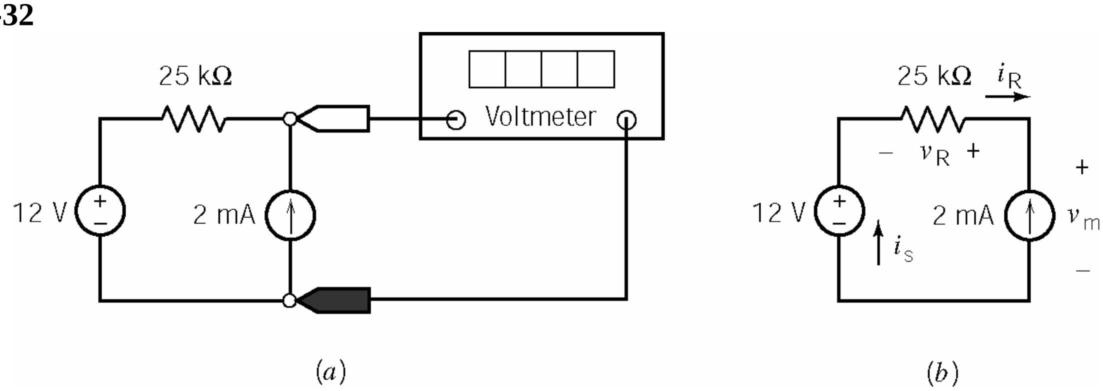
$$R_{eq} = ((R+4)\,||\,20)+2 = \frac{(R+4)\times 20}{(R+4)+20}+2 = \frac{20\,R+80}{R+24}+2$$

a. $$12 = \frac{20\,R+80}{R+24}+2 \;\Rightarrow\; 10 = \frac{20\,R+80}{R+24} \;\Rightarrow\; R+24 = 2\,R+8 \;\Rightarrow\; R = 16\ \Omega$$

b. $$R_{eq} = \frac{20(14)+80}{14+24}+2 = 11.5\ \Omega$$

(Checked: LNAPDC 9/28/04)

P3.6-32



Replace the ideal voltmeter with the equivalent open circuit and label the voltage measured by the meter. Label the element voltages and currents as shown in (***b***).

Using units of V, A, Ω and W:	Using units of V, mA, kΩ and mW:
a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R = v_m$ and $-i_R = -i_s = 2\times 10^{-3}$ A Ohm's law gives $v_R = -(25\times 10^3)\,i_R$ Then	a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R = v_m$ and $-i_R = -i_s = 2$ mA Ohm's law gives $v_R = -25\,i_R$ Then

<table>
<tr>
<td>

$$v_R = -\left(25\times10^3\right) i_R = -\left(25\times10^3\right)\left(-2\times10^{-3}\right)$$
$$= 50\text{ V}$$

$$v_m = 12 + v_R = 12 + 50 = 62\text{ V}$$

b.) Determine the power supplied by each element.

<table>
<tr><td>voltage source</td><td>$12\left(i_s\right) = -12\left(-2\times10^{-3}\right)$
$= -24\times10^{-3}\text{ W}$</td></tr>
<tr><td>current source</td><td>$62\left(2\times10^{-3}\right) = 124\times10^{-3}\text{ W}$</td></tr>
<tr><td>resistor</td><td>$v_R\, i_R = 50\left(-2\times10^{-3}\right)$
$= -100\times10^{-3}\text{ W}$</td></tr>
<tr><td>total</td><td>0</td></tr>
</table>

</td>
<td>

$$v_R = -25\, i_R = -25\left(-2\right) = 50\text{ V}$$

$$v_m = 12 + v_R = 12 + 50 = 62\text{ V}$$

b.) Determine the power supplied by each element.

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<tr><td>voltage source</td><td>$12\left(i_s\right) = -12\left(-2\right)$
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<tr><td>total</td><td>0</td></tr>
</table>

</td>
</tr>
</table>

P3.6-33

$$12 + \frac{40\times10}{40+10} + 4 = 12\ \Omega$$

P3.6-34

$$\frac{\left(60+60+60\right)\times60}{\left(60+60+60\right)+60} = 45\ \Omega$$

Section 3-8 How Can We Check …

P3.8-1

(a)

$$7+(-3)=4 \qquad \text{(node } a\text{)}$$
$$4+(-2)=2 \qquad \text{(node } b\text{)}$$
$$-5=-2+(-3) \qquad \text{(node } c\text{)}$$

(b)

$$-1-(-6)+(-8)+3=0 \qquad \text{(loop } a\text{-}b\text{-}d\text{-}c\text{-}a\text{)}$$
$$-1-2-(-8)-5=0 \qquad \text{(loop } a\text{-}b\text{-}c\text{-}d\text{-}a\text{)}$$

The given currents and voltages satisfy these five Kirchhoff's laws equations.

***P3.8-2**

(a)
$$i=\frac{v_s}{R_1+R_2}$$

from row 1
$$2.4=\frac{v_s}{R_1}$$

from row 2
$$1.2=\frac{v_s}{R_1+10}$$

so

$$2.4R_1=v_s=1.2(R_1+10) \quad \Rightarrow \quad R_1=10\ \Omega$$

then

$$v_s=2.4(10)=24\text{ V}$$

(b)
$$i=\frac{24}{10+R_2} \quad \text{and} \quad v=\frac{24R_2}{10+R_2}$$

When R_2 = 20 Ω then $i=\frac{24}{30}=0.8\text{ A}$ and $v=\frac{480}{30}=16\text{ V}$.

When R_2 = 30 Ω then $v=\frac{720}{40}=18\text{ V}$.

When R_2 = 40 Ω the $i=\frac{24}{50}=0.48\text{ A}$.

(c) When R_2 = 30 Ω then $i = \frac{24}{40} = 0.6$ A .

When R_2 = 40 W then $v = \frac{960}{50} = 19.2$ V .

(checked: LNAP 6/21/04)

P3.8-3

(a)
$$i = \frac{R_1}{R_1 + R_2} i_s$$

From row 1
$$\frac{4}{3} = \frac{R_1}{R_1 + 10} i_s \quad \Rightarrow \quad 4R_1 + 40 = 3R_1 i_s$$

From row 2
$$\frac{6}{7} = \frac{R_1}{R_1 + 20} i_s \quad \Rightarrow \quad 6R_1 + 120 = 7R_1 i_s$$

So
$$\frac{4R_1 + 40}{3R_1} = i_s = \frac{6R_1 + 120}{7R_1} \quad \Rightarrow \quad 28R_1 + 280 = 18R_1 + 360 \quad \Rightarrow \quad R_1 = 8\ \Omega$$

Then
$$\frac{4}{3} = \frac{8}{8+10} i_s \quad \Rightarrow \quad i_s = 3\text{ A}$$

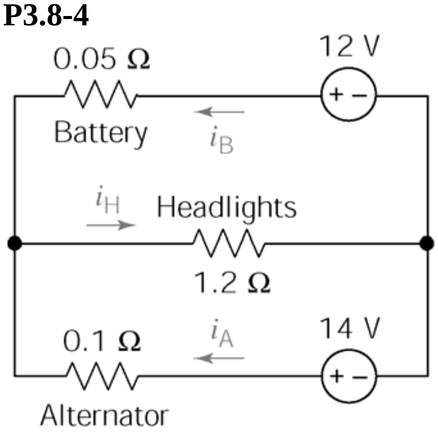
(b)
$$i = \frac{8}{8 + R_2}(3) = \frac{24}{8 + R_2} \quad \text{and} \quad v = R_2\, i = \frac{24R_2}{8 + R_2}$$

When R_2 = 40 Ω then $i = \frac{24}{48} = 0.5$ A and $v = \frac{960}{48} = 20$ V . These are the values in the table so tabulated data is consistent.

(c) When R_2 = 80 Ω then $i = \frac{24}{88} = \frac{3}{11}$ A and $v = \frac{24(80)}{88} = \frac{240}{11}$ V .

(checked: LNAP 6/21/04)

P3.8-4



KVL bottom loop: $-14+0.1i_A+1.2i_H=0$

KVL right loop: $-12+0.05i_B+1.2i_H = 0$

KCL at left node: $i_A+i_B=i_H$

This alone shows the reported results were incorrect.

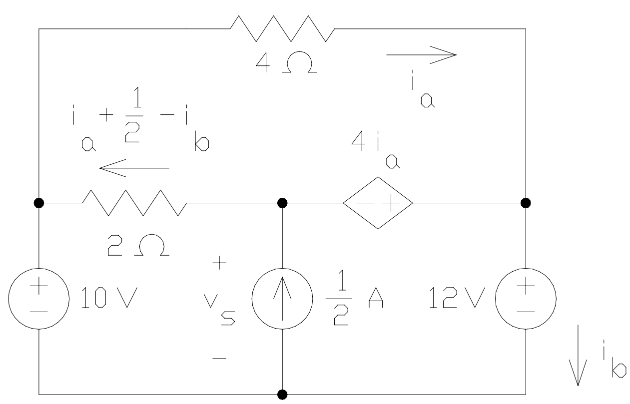
Solving the three above equations yields:

$\underline{i_A=16.8\text{ A}}$ $\underline{i_H=10.3\text{ A}}$

$\underline{i_B=-6.49\text{ A}}$

$\therefore$ Reported values were incorrect.

P3.8-5



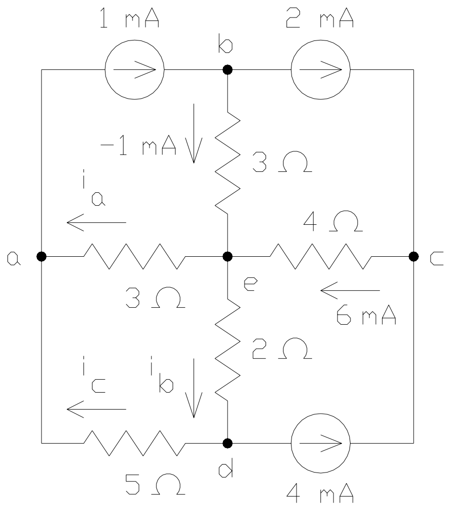
Top mesh: $0=4\,i_a+4\,i_a+2\left(i_a+\frac{1}{2}-i_b\right)=10\left(-0.5\right)+1-2\left(-2\right)$

Lower left mesh: $v_s=10+2\left(i_a+0.5-i_b\right)=10+2\left(2\right)=14\text{ V}$

Lower right mesh: $v_s+4\,i_a=12 \quad\Rightarrow\quad v_s=12-4\,(-0.5)=14\text{ V}$

The KVL equations are satisfied so the analysis is correct.

P3.8-6
Apply KCL at nodes b and c to get:



KCL equations:

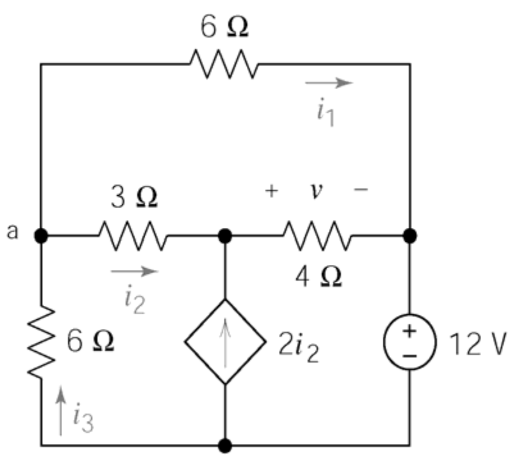
Node e: $-1+6=0.5+4.5$

Node a: $0.5+i_c=-1 \;\Rightarrow\; i_c=-1.5 \text{ mA}$

Node d: $i_c+4=4.5 \;\Rightarrow\; i_c=0.5 \text{ mA}$

That's a contradiction. The given values of i_a and i_b are not correct.

P3.8-7



KCL at node a: $i_3=i_1+i_2$

$$-1.167=-0.833+(-0.333)$$

$$-1.167=-1.166 \quad \text{OK}$$

KVL loop consisting of the vertical 6 Ω resistor, the 3 Ω and4Ω resistors, and the voltage source:

$$6i_3+3i_2+v+12=0$$

yields $\underline{v=-4.0 \text{ V}}$ $\underline{\text{not}}$ $v=-2.0$ V